Section 2.2

Multi-Step Equations

Math 081

DIRECTIONS

As you work through this Learning Guide, you should:

- read carefully
- take notes
- do the **PRACTICE** problems on your own
- check your answers to the **PRACTICE** problems
- watch the videos by clicking on the 🎥 icons

IMPORTANT: Get help if you don't understand a topic.

Quick Links:

Use the links below to go directly to a topic.

Introduction to Solving Multi-Step Equations

In this section you will learn to solve equations that require more than two steps. These equations are called ***multi-step equations.***

The process for solving multi-step equations is described below and on the next slide.

SOLVING MULTI-STEP EQUATIONS

STEPS:

1. Clear parentheses.

   HOW? Use the Distributive Property.

2. Simplify each side of the equation.

   HOW? Combine like terms on each side.

(continued on next slide)

SOLVING MULTI-STEP EQUATIONS (continued)

3. Get the equation to contain just ONE variable term.

   HOW? If there are two variable terms, use inverse operations (+ or −) to eliminate either one.

4. Get the variable term ALONE on one side of the equation.

   HOW? Use inverse operations (+ or −) to eliminate the constant from the side with the variable term.

5. Get the variable alone on one side of the equation.

   HOW? Use inverse operations (× or ÷) to eliminate the coefficient in front of the variable.

IMPORTANT:

Perform the same operation on both sides of the equation.

6. Check the answer.

   HOW? In the original equation, replace the variable(s) with the answer to see if it produces a true statement when simplified.

EXAMPLE: Solve the equation.

1. $4 + 5x - 3x = 5 - 9$

Parentheses: There are no parentheses.

$4 + 5x - 3x = 5 - 9$

Like Terms: On the left side of the equal sign, combine like terms $5x - 3x$. On the right side of the equal sign, combine like terms $5 - 9$.

$4 + 2x = -4$

ONE Variable Term: There is just ONE variable term in the equation: $2x$

$4 + 2x = -4$
$-4 \quad -4$
$2x = -8$

Variable Term ALONE: To get $2x$ alone on the left side of the equal sign, subtract 4 from both sides of the equation.

$$\frac{2x}{2} = \frac{-8}{2}$$

$x = -4$

Variable Alone: To get x alone on the left side of the equal sign, divide by 2 on both sides of the equation.

EXAMPLE: Solve the equation.

2. $8x - 8 = 2(7x - 25)$

Parentheses: Use the Distributive Property to clear the parentheses.

$8x - 8 = 14x - 50$

Like Terms: There are no like terms to combine on either side of the equation.

$8x - 8 = 14x - 50$

$-8x \qquad -8x$

$-8 = 6x - 50$

ONE Variable Term: The equation has two variable terms: $8x$ and $14x$. We will use inverse operations to remove one of those terms. We decide to remove $8x$, so we perform the ***inverse*** and $-8x$ from both sides of the equation.

After simplifying, there is just ONE variable term: $6x$

$-8 = 6x - 50$

$+50 \qquad +50$

$42 = 6x$

Variable Term ALONE: To get $6x$ alone on the right side of the equal sign, add 50 to both sides of the equation.

$\frac{42}{6} = \frac{6x}{6}$

$7 = x$

Variable Alone: To get x alone on the right side of the equal sign, divide by 6 on both sides of the equation.

EXAMPLE: Solve the equation.

3. $3(x-2)-5 = 7x+3$

Parentheses: Use the Distributive Property to clear the parentheses.

$3x-6-5 = 7x+3$

Like Terms: On the left side of the equal sign, combine like terms –6 – 5.

$3x-11 = 7x+3$

$-3x \qquad -3x$

ONE Variable Term: The equation has two variable terms: $3x$ and $7x$. We will use inverse operations to remove one of those terms. We decide to remove $3x$, so we perform the ***inverse*** and $-3x$ from both sides of the equation.

$-11 = 4x+3$

After simplifying, there is just ONE variable term: $4x$

$-11 = 4x+3$

$-3 \qquad -3$

Variable Term ALONE: To get $4x$ alone on the right side of the equal sign, subtract 3 from both sides of the equation.

$-14 = 4x$

$\frac{-14}{4} = \frac{4x}{4}$

Variable Alone: To get x alone on the right side of the equal sign, divide by 4 on both sides of the equation. Simplify the fraction on the left side.

$-\frac{7}{2} = x$

SPECIAL CASES

Sometimes the solution of an equation involves one of these two special cases:

SPECIAL CASES	
No Solution	**All Real Numbers**
When solving an equation: ▪ if the simplified equation is a **False** statement (*like* $5 = 9$), ▪ then the original equation has **No Solution** (*represented by the symbol* $\varnothing$). Note: This means that there is no number that can replace the variable in the original equation to make it true.	When solving an equation: ▪ if the simplified equation is a **True** statement (*like* $5 = 5$), ▪ then the original equation has **All Real Numbers** as the solution. Note: This means that every real number can replace the variable in the original equation to make it true.

EXAMPLE: Solve the equation.

1. $7x-(4x-2)=3x+11$

$7x-1(4x-2)=3x+11$

Parentheses: Use the Distributive Property to clear the parentheses.
It may help to insert a 1 in front of the parentheses in order to distribute –1.

$7x-4x+2=3x+11$

Like Terms: On the left side of the equal sign, combine like terms $7x-4x$.

$3x+2=3x+11$

ONE Variable Term: The equation has two variable terms: $3x$ on the left side of the equation and another $3x$ on the right side of the equation.

$-3x \quad -3x$

$2=11$

To remove $3x$, we perform the ***inverse*** and $-3x$ from both sides.

Simplify the left side of the equation: $3x-3x$ is 0. Just the 2 remains.
Simplify the right side of the equation: $3x-3x$ is 0. Just the 11 remains.

$2=11$

The resulting equation has no variable, and has different numbers on each side.
This is a false statement. It means that the original equation has No Solution.

Answer: $\varnothing$

We use the symbol $\varnothing$ when there is No Solution.

EXAMPLE: Solve the equation.

2. $4(3x-8) = 7x+5x-32$ Parentheses: Use the Distributive Property to clear the parentheses.

$12x-32 = 7x+5x-32$ Like Terms: On the right side of the equal sign, combine like terms $7x + 5x$.

$12x-32 = 12x-32$ ONE Variable Term: The equation has two variable terms: $12x$ on the left side of the equation and another $12x$ on the right side of the equation.

$\cancel{12x} - 32 = \cancel{12x} - 32$
$-\cancel{12x} \quad\quad -\cancel{12x}$ To remove $12x$, we perform the ***inverse*** and $-12x$ from both sides.

$-32 = -32$ Simplify each side. Just -32 remains on each side.

$-32 = -32$ The resulting equation has no variable, and the same number is on both sides. This is a true statement. It means that every real number is a solution of the original equation.

Solution: All Real Numbers

PRACTICE: Solve the equation. *Be sure to write out all the algebra steps.*

1. $2a-1+4=5$

2. $7x-11=2x+5$

3. $2(3-x)+4x=-3+7$

PRACTICE: Answers Solve the equation. *Be sure to write out all the algebra steps.*

1. $2a-1+4=5$ $a=1$

2. $7x-11=2x+5$ $x=\frac{16}{5}$

3. $2(3-x)+4x=-3+7$ $x=-1$

PRACTICE: Solve the equation. *Be sure to write out all the algebra steps.*

4. $3x+2(3x+1)=9x-8$

5. $5-(3x-6)=-2x+1$

6. $2(2x-1)-3=-4(x+4)$

PRACTICE: Answers Solve the equation. *Be sure to write out all the algebra steps.*

4. $3x+2(3x+1)=9x-8$ $\varnothing$

5. $5-(3x-6)=-2x+1$ $x=10$

6. $2(2x-1)-3=-4(x+4)$ $x=-\frac{11}{8}$

SOLVING EQUATIONS WITH RATIONAL NUMBERS

SOLVING EQUATIONS WITH RATIONAL NUMBERS
If an equation contains one or more fractions, eliminate the fractions first. Do this by multiplying **every term** on both sides of the equation by the *Least Common Denominator (LCD)*.

EXAMPLE: Solve the equation.

1. Solve $-\frac{2}{7}x = 4$

$-\frac{2}{7}x = 4$ Determine the LCD:
There is only one fraction, and its denominator is 7. So, the LCD is **7**.

$7\left(-\frac{2}{7}x\right) = 7(4)$ Multiply both sides of the equation by **7**, the LCD.

$\frac{\mathbf{1}\,\cancel{7}}{1}\left(-\frac{2}{\cancel{7}_{\mathbf{1}}}x\right) = 7(4)$ On the left side of the equation, write 7 as a fraction with a denominator of 1, then divide out common factors.

Perform the multiplication on each side of the equation.

$-2x = 28$ Notice that multiplying by the LCD eliminated the fractions.

$\frac{\cancel{-2}x}{\cancel{-2}} = \frac{28}{-2}$ To get the variable alone, divide by –2 on both sides of the equation.

$x = -14$ This is the solution.

EXAMPLE: Solve the equation.

2. Solve $2x - \frac{8}{5} = \frac{16}{15}$

$$2x - \frac{8}{5} = \frac{16}{15}$$

$$15(2x) - 15\left(\frac{8}{5}\right) = 15\left(\frac{16}{15}\right)$$

$$\underbrace{15(2x)} - \underbrace{\frac{{}^{3}\cancel{15}}{1}\left(\frac{8}{\cancel{5}_{1}}\right)} = \underbrace{\frac{{}^{1}\cancel{15}}{1}\left(\frac{16}{\cancel{15}_{1}}\right)}$$

$$30x - 24 = 16$$

$$\cancel{+24} \qquad +24$$

$$30x = 40$$

$$\frac{\cancel{30}x}{\cancel{30}} = \frac{40}{30}$$

$$x = \frac{4}{3}$$

Determine the LCD: The denominators of the fractions are 5 and 15. The LCD of 5 and 15 is **15**.

Multiply every term on both sides of the equation by **15**, the LCD.

Write integers as fractions with a denominator of 1, then divide out common factors.

Perform the multiplications on each side.

Multiplying by the LCD eliminated the fractions.

To get the variable term $30x$ alone, add 24 to both sides of the equation.

To get x alone, divide by 30 on both sides of the equation. Then simplify the fraction.

This is the solution.

EXAMPLE: Solve the equation.

3. Solve $\frac{3}{4}x-\frac{1}{5}=\frac{1}{2}x+\frac{3}{4}$

$$\frac{3}{4}x - \frac{1}{5} = \frac{1}{2}x + \frac{3}{4}$$

Determine the LCD: The denominators of the fractions are 2, 4, and 5. The LCD is **20**.

$$20\left(\frac{3}{4}x\right) - 20\left(\frac{1}{5}\right) = 20\left(\frac{1}{2}x\right) + 20\left(\frac{3}{4}\right)$$

Multiply every term on both sides of the equation by **20**, the LCD.

$$\frac{\overset{5}{\cancel{20}}}{1}\left(\frac{3}{\cancel{4}_{1}}x\right) - \frac{\overset{4}{\cancel{20}}}{1}\left(\frac{1}{\cancel{5}_{1}}\right) = \frac{\overset{10}{\cancel{20}}}{1}\left(\frac{1}{\cancel{2}_{1}}x\right) + \frac{\overset{5}{\cancel{20}}}{1}\left(\frac{3}{\cancel{4}_{1}}\right)$$

Write integers as fractions with a denominator of 1, then divide out common factors.

$$15x - 4 = 10x + 15$$

Perform the multiplications on each side of the equation to eliminate the fractions.

There are two variable terms: $15x$ and $10x$.

$$\begin{array}{rcl} -10x & & -10x \\ \hline 5x & = & 15 \end{array}$$

To get just one variable term, subtract $10x$ from both sides.

After simplifying, the equation contains just one variable term: $5x$

$$\begin{array}{rcl} \cancel{-4} & = & 15 \\ \cancel{+4} & & +\ 4 \\ \hline 5x & = & 19 \end{array}$$

To get the variable term $5x$ alone, add 4 to both sides of the equation.

$$\frac{\cancel{5}x}{\cancel{5}} = \frac{19}{5}$$

To get x alone, divide by 5 on both sides.

$$x = \frac{19}{5}$$

This is the solution.

PRACTICE: Solve the equation. *Be sure to write out all the algebra steps.*

1. $\frac{3}{4}x = 12$

2. $-\frac{8}{9} = x - 1$

3. $-\frac{4}{7}x = \frac{2}{9}$

PRACTICE: Answers Solve the equation. *Be sure to write out all the algebra steps.*

1. $\frac{3}{4}x = 12$ $x = 16$ 

2. $-\frac{8}{9} = x - 1$ $x = \frac{1}{9}$

3. $-\frac{4}{7}x = \frac{2}{9}$ $x = -\frac{7}{18}$ 

PRACTICE: Solve the equation. *Be sure to write out all the algebra steps.*

4. $\frac{3}{5}+x=\frac{2}{3}x$

5. $\frac{3}{5}x-\frac{1}{6}=\frac{1}{2}x$

6. $\frac{4}{3}x-\frac{1}{2}=\frac{1}{4}x+3$

PRACTICE: Answers Solve the equation. *Be sure to write out all the algebra steps.*

4. $\frac{3}{5}+x=\frac{2}{3}x$ $x=-\frac{9}{5}$ 

5. $\frac{3}{5}x-\frac{1}{6}=\frac{1}{2}x$ $x=\frac{5}{3}$

6. $\frac{4}{3}x-\frac{1}{2}=\frac{1}{4}x+3$ $x=\frac{42}{13}$ 

Solving Application Problems

SOLVING WORD PROBLEMS

1. Read the problem.
   - Read until you understand it.
   - Highlight important information.
   - Draw a picture if helpful.

2. Define a variable.
   - Identify what you are asked to find.
   - Choose a variable to represent the unknown quantity.

3. Write an algebraic equation.
   - Break the sentences down and translate one part at a time into math.
   - Use key words to identify math operations.
   - Look for relationships among quantities.

(continued on next slide)

SOLVING WORD PROBLEMS (continued)

4. Solve the equation.
   - Perform the algebraic steps to isolate the variable.

5. Check the answer.
   - Substitute the answer in the original equation.
   - Be sure the answer makes sense in the context of the problem.

6. State the answer.
   - Write a phrase or sentence giving the answer.

EXAMPLE: Write an algebraic equation for the word problem, then solve the problem to answer the question.

1. *A medical center is planning to hire a total of 54 nurses and CNAs (Certified Nursing Assistants). If the center needs twice as many CNAs as nurses, how many of each should they hire?*

Variable: n = the number of nurses to hire

Equation:

$$\underbrace{\#\text{ of nurses}}_{n} + \underbrace{\#\text{ of CNAs}}_{2n} = \underbrace{\text{total }\#\text{ nurses and CNAs}}_{54}$$

Solve:

$$n + 2n = 54$$ Combine like terms on the left side of the equation.

$$3n = 54$$

$$\frac{3n}{3} = \frac{54}{3}$$ To get n alone, divide by 3 on both sides of the equation.

$$n = 18$$

Answer:

$$\#\text{ of nurses} = n = 18$$
$$\#\text{ of CNAs} = 2n = 2(18) = 36$$

The center should hire 18 nurses and 36 CNAs.

EXAMPLE: Write an algebraic equation for the word problem, then solve the problem to answer the question.

2. *The auto repair shop took 2.5 hours to repair Victoria's car. The cost of the parts needed was $93, and the total bill was $248. What is the repair shop's charge per hour for labor?*

Variable: x = hourly charge for labor

Equation:

$$\underbrace{\text{Cost for Parts}}_{93} + \underbrace{\text{Cost for Labor}}_{\text{(\# of hours) (hourly charge)}} = \underbrace{\text{Total Cost}}_{248}$$

$$93 + (2.5)(x) = 248$$

Solve:

$$93 + 2.5x = 248$$

$$-93 \qquad -93$$

To get the variable term $2.5x$ alone, subtract 93 from both sides of the equation.

$$\frac{2.5x}{2.5} = \frac{155}{2.5}$$

To get the variable x alone, divide by 2.5 on both sides of the equation.

$$x = 62$$

Answer: The repair shop's charge for labor is $62 per hour.

EXAMPLE: Write an algebraic equation for the word problem, then solve the problem to answer the question.

3. *Your grades on your first three math tests were 81, 76, and 78. You have one more test to take. What grade do you need on that last test in order to get an 80 average?*

Variable: x = the grade you need on the last test

Equation: $$\text{Average} = \frac{\text{Sum of Test Grades}}{\text{\# of Tests}}$$

$$\text{Average} = \frac{\text{Test 1} + \text{Test 2} + \text{Test 3} + \text{Test 4}}{4}$$

$$80 = \frac{81+76+78+x}{4}$$

Solve:

$$80 = \frac{81+76+78+x}{4}$$ The LCD is 4.

$$4(80) = 4\left(\frac{81+76+78+x}{4}\right)$$ Multiply both sides of the equation by **4**.

$$4(80) = \frac{\cancel{4}}{1}\left(\frac{81+76+78+x}{\cancel{4}}\right)$$ Multiplying by the LCD eliminates the fractions.

$$320 = 81+76+78+x$$ Combine like terms on the right side of the equation.

$$320 = 235 + x$$

$$-235 \quad -235$$ To get x alone, subtract 235 from both sides.

$$85 = x$$

Answer: You will need to score 85 on the last test.

PRACTICE: Answers Solve the equation. *Be sure to write out all the algebra steps.*

1. Kevin plans to buy a used car. The monthly payments will be $259. If Kevin earns $9.25 per hour, how many hours must Kevin work each month to afford his car payment?

2. The bottle of juice says that it has three times as much orange juice as apple juice. If the bottle contains 64 fluid ounces, how many ounces of orange juice does it contain?

PRACTICE: Answers Solve the equation. *Be sure to write out all the algebra steps.*

1. Kevin plans to buy a used car. The monthly payments will be \$259. If Kevin earns \$9.25 per hour, how many hours must Kevin work each month to afford his car payment?

Equation: $9.25h = 259$

Solution: 28 hours

2. The bottle of juice says that it has three times as much orange juice as apple juice. If the bottle contains 64 fluid ounces, how many ounces of orange juice does it contain?

Equation: $a + 3a = 64$

Solution: 48 fluid ounces

PRACTICE: Answers Solve the equation. *Be sure to write out all the algebra steps.*

3. Jim is planning to take some classes at the community college. Tuition is $110 per credit, plus $60 in fees. Jim has saved $1050 to take classes. How many credits can Jim afford to take with the money he saved?

4. Sara has $75 to spend on clothes. She wants to buy a pair of jeans that cost $27 and spend the rest on t-shirts. Each t-shirt costs $8. How many t-shirts can Sara afford to buy?

PRACTICE: Answers Solve the equation. *Be sure to write out all the algebra steps.*

3. Jim is planning to take some classes at the community college. Tuition is \$110 per credit, plus \$60 in fees. Jim has saved \$1050 to take classes. How many credits can Jim afford to take with the money he saved?

Equation: $110c + 60 = 1050$

Solution: 9 credits

4. Sara has \$75 to spend on clothes. She wants to buy a pair of jeans that cost \$27 and spend the rest on t-shirts. Each t-shirt costs \$8. How many t-shirts can Sara afford to buy?

Equation: $27 + 8s = 75$

Solution: 6 shirts

This is the end of the PowerPoint Learning Guide for Section 2.2.

Return to Section 2.2 of the Brightspace course to:

- study the Summary
- complete the Exercise Set