Chapter 2: Systems of Equations

Section 2.1: Solving Systems of Equations by Graphing

Objective: Solve systems of equations by graphing and identifying the point of intersection.

We have solved equations like $3x-4=11$ by adding 4 to both sides and then dividing by 3 (solution is $x=5$). We also have methods to solve equations with more than one variable in them. It turns out that to solve for more than one variable we will need the same number of equations as variables. For example, to solve for two variables such as x and y we will need two equations. When we have several equations we are using to solve, we call the equations a **system of equations**. When solving a system of equations we are looking for a solution that works for all of these equations. In this discussion, we will limit ourselves to solving two equations with two unknowns. This solution is usually given as an ordered pair (x, y). The following example illustrates a solution working in both equations.

Example 1.

Show $(2,1)$ is the solution to the system $\begin{aligned} 3x-y&=5 \\ x+y&=3 \end{aligned}$

$(2,1)$	Identify x and y from the ordered pair
$x=2,\ y=1$	Plug these values into each equation
$3(2)-(1)=5$	First equation
$6-1=5$	Evaluate
$5=5$	True
$(2)+(1)=3$	Second equation, evaluate
$3=3$	True

As we found a true statement for both equations we know $(2,1)$ is the solution to the system. It is in fact the only combination of numbers that works in both equations. In this lesson we will be working to find this point given the equations. It seems to follow that if we use points to describe the solution, we can use graphs to find the solutions.

If the graph of a line is a picture of all the solutions, we can graph two lines on the same coordinate plane to see the solutions of both equations. We are interested in the point that is a solution for both lines, this would be where the lines intersect! If we can find the intersection of the lines we have found the solution that works in both equations.

Example 2.

Solve the system of equations by graphing: $y=-\frac{1}{2}x+3$
$y=\frac{3}{4}x-2$

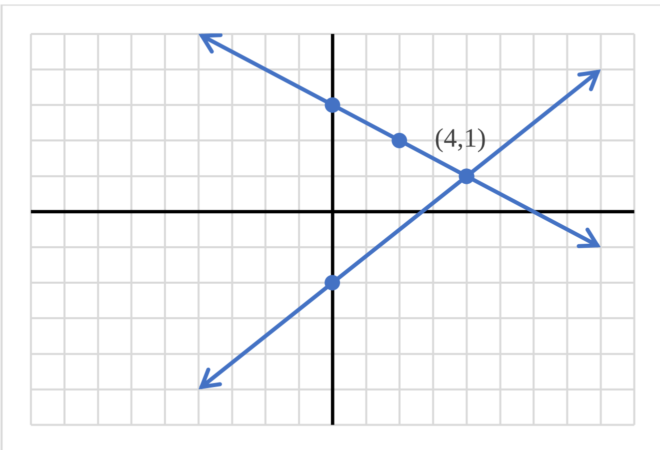
$$y=-\frac{1}{2}x+3$$ To graph we identify slopes and y-intercepts

$$y=\frac{3}{4}x-2$$

$$\text{First: } m=-\frac{1}{2},\ b=3$$

Now we can graph both lines on the same plane.

$$\text{Second: } m=\frac{3}{4},\ b=-2$$



To graph each equation, we start at the y-intercept and use the slope $\frac{\text{rise}}{\text{run}}$ to get the next point and connect the dots.

Remember a negative slope is downhill!

Find the intersection point, $(4,1)$

$(4,1)$ Our Solution

Often the equations will not be in slope-intercept form. We can solve both equations for y first to put the equation in slope-intercept form.

Example 3.

Solve the system of equations by graphing: $6x-3y=-9$
$2x+2y=-6$

$$6x-3y=-9$$
$$2x+2y=-6$$ Solve each equation for y

$$\begin{array}{rr} 6x-3y=-9 & 2x+2y=-6 \\ \underline{-6x \qquad -6x} & \underline{-2x \qquad -2x} \\ -3y=-6x-9 & 2y=-2x-6 \end{array}$$

Subtract x terms
Put x terms first

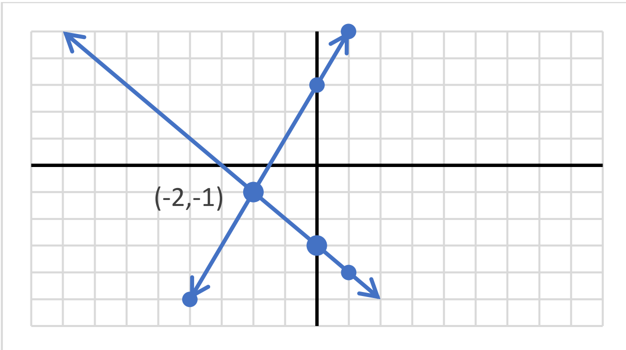
$$\frac{-3y}{-3}=\frac{-6x}{-3}-\frac{9}{-3} \qquad \frac{2y}{2}=\frac{-2x}{2}-\frac{6}{2}$$ Divide by coefficient of y

$y = 2x + 3 \qquad y = -x - 3$ Identify slope and *y*-intercepts

First: $m = \frac{2}{1}, b = 3$

Second: $m = -\frac{1}{1}, b = -3$

Now we can graph both lines on the same plane



To graph each equation, we start at the *y*-intercept and use the slope $\frac{\text{rise}}{\text{run}}$ to get the next point and connect the dots.

Remember a negative slope is downhill!

Find the intersection point, $(-2, -1)$

$(-2, -1)$ Our Solution

As we are graphing our lines, it is possible to have one of two unexpected results. These are shown and discussed in the next two examples.

Example 4.

Solve the system of equations by graphing: $y = \frac{3}{2}x - 4$, $y = \frac{3}{2}x + 1$

$y = \frac{3}{2}x - 4$

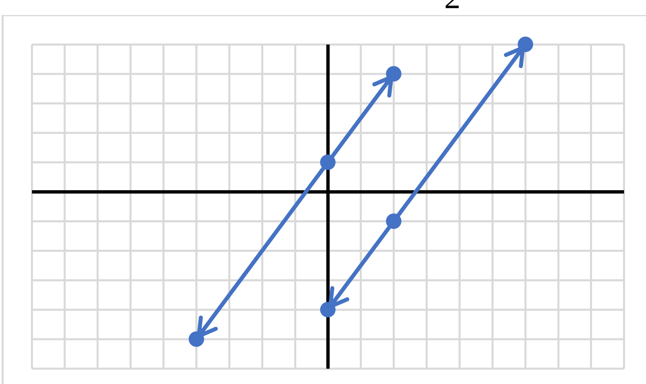
$y = \frac{3}{2}x + 1$

Identify slope and *y*-intercept of each equation

First: $m = \frac{3}{2}, b = -4$

Second: $m = \frac{3}{2}, b = 1$

Now we can graph both lines on the same plane



To graph each equation, we start at the *y*-intercept and use the slope $\frac{\text{rise}}{\text{run}}$ to get the next point and connect the dots.

The two lines do not intersect! They are parallel! If the lines do not intersect we know that there is no point that works in both equations. There is no solution.

$\varnothing$ No Solution

You can graph lines using intercepts as well.

Example 5.

Solve the system of equations by graphing: $\begin{aligned} 2x - 6y &= 12 \\ 3x - 9y &= 18 \end{aligned}$

$2x - 6y = 12$ Let us graph using intercepts.

$2x - 6(0) = 12$ x-intercept (let $y = 0$)

$2x = 12$

$x = 6$

$(6, 0)$ Our x-intercept

$2(0) - 6y = 12$

$-6y = 12$

$y = -2$

$(0, -2)$ Our y-intercept

$3x - 9y = 18$ Let us graph using intercepts again.

$3x - 9(0) = 18$ x-intercept (let $y = 0$)

$3x = 18$

$x = 6$

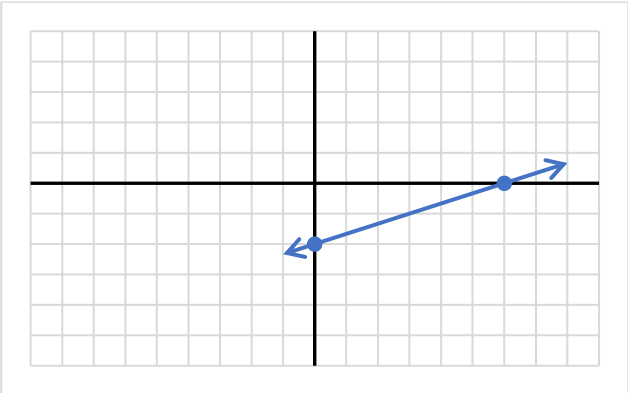
$(6, 0)$ Our x-intercept

$3(0) - 9y = 18$

$-9y = 18$

$y = -2$

$(0, -2)$ Our y-intercept



Both equations are the same line! As one line is directly on top of the other line, we can say that the lines "intersect" at all the points! Here we say we have infinite solutions.

World View Note: The Babylonians were the first to work with systems of equations with two variables. However, their work with systems was quickly passed by the Greeks who would solve systems of equations with three or four variables and, around 300 AD, developed methods for solving systems with any number of unknowns!

2.1 Practice

Solve each system of equations by graphing.

1) $y = -x + 1$
$y = -5x - 3$

2) $y = -\frac{5}{4}x - 2$
$y = -\frac{1}{4}x + 2$

3) $y = -3$
$y = -x - 4$

4) $y = -x - 2$
$y = \frac{2}{3}x + 3$

5) $y = -\frac{3}{4}x + 1$
$y = -\frac{3}{4}x + 2$

6) $y = 2x + 2$
$y = -x - 4$

7) $y = \frac{1}{3}x + 2$
$y = -\frac{5}{3}x - 4$

8) $y = 2x - 4$
$y = \frac{1}{2}x + 2$

9) $y = \frac{5}{3}x + 4$
$y = -\frac{2}{3}x - 3$

10) $y = \frac{1}{2}x + 4$
$y = \frac{1}{2}x + 1$

11) $x + 3y = -9$
$5x + 3y = 3$

12) $x + 4y = -12$
$2x + y = 4$

13) $\begin{array}{l} x - y = 4 \\ 2x + y = -1 \end{array}$

14) $\begin{array}{l} 6x + y = -3 \\ x + y = 2 \end{array}$

15) $\begin{array}{l} 2x + 3y = -6 \\ 2x + y = 2 \end{array}$

16) $\begin{array}{l} 3x + 2y = 2 \\ 3x + 2y = -6 \end{array}$

17) $\begin{array}{l} 2x + y = 2 \\ x - y = 4 \end{array}$

18) $\begin{array}{l} x + 2y = 6 \\ 5x - 4y = 16 \end{array}$

19) $\begin{array}{l} 2x + y = -2 \\ x + 3y = 9 \end{array}$

20) $\begin{array}{l} x - y = 3 \\ 5x + 2y = 8 \end{array}$

21) $\begin{array}{l} 2y = 4x + 6 \\ y = 2x + 3 \end{array}$

22) $\begin{array}{l} -2y + x = 4 \\ 2 = -x + \frac{1}{2}y \end{array}$

23) $\begin{array}{l} 2x - y = -1 \\ 0 = -2x - y - 3 \end{array}$

24) $\begin{array}{l} -2y = -4 - x \\ -2y = -5x + 4 \end{array}$

25) $\begin{array}{l} 3 + y = -x \\ -4 - 6x = -y \end{array}$

26) $\begin{array}{l} 16 = -x - 4y \\ -2x = -4 - 4y \end{array}$

27) $\begin{array}{l} -y + 7x = 4 \\ -y - 3 + 7x = 0 \end{array}$

28) $\begin{array}{l} -4+y=x \\ x+2=-y \end{array}$

29) $\begin{array}{l} -12+x=4y \\ 12-5x=4y \end{array}$

30) $\begin{array}{l} 12x-3y=9 \\ y=4x-3 \end{array}$

2.1 Answers

1) $(-1, 2)$
2) $(-4, 3)$
3) $(-1, -3)$
4) $(-3, 1)$
5) No Solution
6) $(-2, -2)$
7) $(-3, 1)$
8) $(4, 4)$
9) $(-3, -1)$
10) No Solution
11) $(3, -4)$
12) $(4, -4)$
13) $(1, -3)$
14) $(-1, 3)$
15) $(3, -4)$
16) No Solution
17) $(2, -2)$
18) $(4, 1)$
19) $(-3, 4)$
20) $(2, -1)$
21) Infinite Solutions
22) $(-4, -4)$
23) $(-1, -1)$
24) $(2, 3)$
25) $(-1, -2)$
26) $(-4, -3)$
27) No Solution
28) $(-3, 1)$
29) $(4, -2)$
30) Infinite Solutions

Section 2.2: Solving Systems of Equations by Substitution

Objective: Solve systems of equations using substitution.

Solving a system by graphing has several limitations. First, it requires the graph to be perfectly drawn**.** If the lines are not straight we may arrive at the wrong answer. Second, graphing is not a great method to use if the answer is really large, such as $(100,-75)$, or if the answer contains a decimal that the graph will not help us find, such as $(3.2134,\ 2.17)$. For these reasons we will rarely use graphing to solve our systems. Instead, an algebraic approach will be used.

The first algebraic approach is called substitution. We will build the concepts of substitution through several examples, then end with a five-step process to solve problems using this method.

Example 1.

Solve the systems of equations by using substitution: $\begin{array}{l} x=5 \\ y=2x-3 \end{array}$

$x=5$ $y=2x-3$	We already know $x=5$, substitute this into the other equation
$y=2(\mathbf{5})-3$	Evaluate, multiply first
$y=10-3$	Subtract
$y=7$	We now also have y
$(5,7)$	Our Solution

When we know what one variable equals we can plug that value (or expression) in for the variable in the other equation. It is very important that when we substitute, the substituted value goes in parentheses. The reason for this is shown in the next example.

Example 2.

Solve the systems of equations by using substitution: $\begin{array}{l} 2x-3y=7 \\ y=3x-7 \end{array}$

$2x-3y=7$ $y=3x-7$	We know $y=3x-7$; substitute this into the other equation
$2x-3(\mathbf{3x-7})=7$	Solve this equation, distributing -3 first
$2x-9x+21=7$	Combine like terms $2x-9x$
$-7x+21=7$	Subtract 21

$$
\begin{array}{rl}
-7x+21=7 & \\
\mathbf{-21}\quad\mathbf{-21} & \\ \hline
-7x=-14 & \\
\dfrac{-7x}{\mathbf{-7}}=\dfrac{-14}{\mathbf{-7}} & \text{Divide by } -7 \\
x=2 & \text{We now have our } x\text{; plug into the } y= \text{ equation to find } y \\
y=3(\mathbf{2})-7 & \text{Evaluate, multiply first} \\
y=6-7 & \text{Subtract} \\
y=-1 & \text{We now also have } y \\
(2,-1) & \text{Our Solution}
\end{array}
$$

By using the entire expression $3x-7$ to replace y in the other equation we were able to reduce the system to a single linear equation, which we can easily solve for our first variable. However, the lone variable (a variable without a coefficient) is not always alone on one side of the equation. If this happens we can isolate it by solving for the lone variable.

Example 3.

Solve the systems of equations by using substitution: $\begin{array}{l} 3x+2y=1 \\ x-5y=6 \end{array}$

$$
\begin{array}{rl}
3x+2y=1 & \text{Lone variable is } x\text{; isolate by adding } 5y \text{ to both sides.} \\
x-5y=6 & \\
\mathbf{+5y}\quad\mathbf{+5y} & \\ \hline
x=6+5y & \text{Substitute this into the untouched equation} \\
3(\mathbf{6+5y})+2y=1 & \text{Solve this equation, distributing } 3 \text{ first} \\
18+15y+2y=1 & \text{Combine like terms } 15y+2y \\
18+17y=1 & \\
\mathbf{-18}\qquad\quad\mathbf{-18} & \\ \hline
17y=-17 & \\
\dfrac{17y}{\mathbf{17}}=\dfrac{-17}{\mathbf{17}} & \text{Divide both sides by } 17 \\
y=-1 & \text{We have our } y\text{; plug this into the } x= \text{ equation to find } x \\
x=6+5(-1) & \text{Evaluate, multiply first} \\
x=6-5 & \text{Subtract} \\
x=1 & \text{We now also have } x \\
(1,-1) & \text{Our Solution}
\end{array}
$$

The process in the previous example is how we will solve problems using substitution. This process is described and illustrated in the following table which lists the five steps to solving by substitution.

Problem	$4x-2y=2$ $2x+y=-5$
1. Find the lone variable	Second Equation, y $2x+\mathbf{y}=-5$
2. Solve for the lone variable	$-2x \quad -2x$ $\mathbf{y = -5 - 2x}$
3. Substitute into the untouched equation	$4x-2(\mathbf{-5-2x})=2$
4. Solve	$4x+10+4x=2$ $8x+10=2$ $-10 \quad -10$ $\frac{8x}{8}=\frac{-8}{8}$ $x=-1$
5. Plug into lone variable equation and evaluate	$y=-5-2(\mathbf{-1})$ $y=-5+2$ $\mathbf{y=-3}$
Solution	$(-1,-3)$

Sometimes we have several lone variables in a problem. In this case we will have the choice on which lone variable we wish to solve for; either choice will give the same final result.

Example 4.

Solve the system of equations by using substitution.

$\mathbf{x}+y=5$	Find the lone variable: x or y in first, or x in second.
$x-y=-1$	We will chose x in the first
$x+y=5$ $\underline{\quad -\mathbf{y} \quad -\mathbf{y}}$	Solve for the lone variable; subtract y from both sides
$x=5-y$	Plug into the untouched equation, the second equation
$\mathbf{(5-y)}-y=-1$	Solve (parentheses are not needed here); combine like terms
$5-2y=-1$ $\underline{-5 \qquad -5}$ $-2y=-6$	Subtract 5 from both sides
$\frac{-2y}{-2}=\frac{-6}{-2}$	Divide both sides by -2

$y=3$	We have our y!
$x=5-(3)$	Plug into lone variable equation; evaluate
$x=2$	Now we have our x
$(2,3)$	Our Solution

Just as with graphing it is possible to have no solution $\varnothing$ (parallel lines) or infinite solutions (same line) with the substitution method. While we won't have a parallel line or the same line to look at and conclude if it is one or the other, the process takes an interesting turn as shown in the following example.

Example 5.

$\boldsymbol{y}+4=3x$	Find the lone variable, y, in the first equation
$2y-6x=-8$	
$y+4=3x$	Solve for the lone variable; subtract 4 from both sides
$\underline{\quad -4\quad -4}$	
$y=3x-4$	Plug into untouched equation
$2(\boldsymbol{3x-4})-6x=-8$	Solve; first distribute 2 through parentheses grouping
$6x-8-6x=-8$	Combine like terms $6x-6x$
$-8=-8$	Variables are gone! A true statement.
Infinite solutions	Our Solution

Because we had a true statement, and no variables, we know that anything that works in the first equation, will also work in the second equation. However, we do not always end up with a true statement.

Example 6.

$6x-3y=-9$	Find the lone variable, y, in the second equation
$-2x+\boldsymbol{y}=5$	
$-2x+y=5$	Solve for the lone variable; add $2x$ to both sides
$\underline{+2x\qquad +2x}$	
$y=5+2x$	Plug into untouched equation
$6x-3(\boldsymbol{5+2x})=-9$	Solve, first distribute through parentheses grouping
$6x-15-6x=-9$	Combine like terms $6x-6x$
$-15\neq-9$	Variables are gone! A false statement.
No solution $\varnothing$	Our Solution

Because we had a false statement and no variables, we know that no numerical values will work in both equations.

World View Note: French mathematician Rene Descartes wrote a book which included an appendix on geometry. It was in this book that he suggested using letters from the end of the alphabet for unknown values. This is why often we are solving for the variables x, y and z.

One more question needs to be considered, what if there is no lone variable? If there is no lone variable substitution can still work to solve, we will just have to select one variable to solve for and use fractions as we solve.

Example 7.

$5x-6y=-14$	No lone variable
$-2x+4y=12$	We will solve for x in the first equation
$5x-6y=-14$	Solve for our variable, add $6y$ to both sides
$\underline{\quad +\mathbf{6y} \quad +\mathbf{6y}}$	
$\frac{5x}{\mathbf{5}}=\frac{-14}{\mathbf{5}}+\frac{6y}{\mathbf{5}}$	Divide each term by 5
$x=\frac{-14}{5}+\frac{6y}{5}$	Plug into untouched equation
$-2\left(\frac{-14}{5}+\frac{6y}{5}\right)+4y=12$	Solve, distribute through parenthesis
$\frac{28}{5}-\frac{12y}{5}+4y=12$	Clear fractions by multiplying by 5
$\frac{28(\mathbf{5})}{5}-\frac{12y(\mathbf{5})}{5}+4y(\mathbf{5})=12(\mathbf{5})$	Reduce fractions and multiply
$28-12y+20y=60$	Combine like terms $-12y+20y$
$28+8y=60$	Subtract 28 from both sides
$\underline{\mathbf{-28} \qquad \mathbf{-28}}$	
$\frac{8y}{\mathbf{8}}=\frac{32}{\mathbf{8}}$	Divide both sides by 8
$y=4$	We have our y
$x=\frac{-14}{5}+\frac{6(4)}{5}$	Plug into lone variable equation, multiply
$x=\frac{-14}{5}+\frac{24}{5}$	Add fractions
$x=\frac{10}{5}$	Reduce fraction
$x=2$	Now we have our x
$(2,4)$	Our Solution

Using the fractions could make the problem a bit trickier. This is why we have another method for solving systems of equations that will be discussed in another lesson.

2.2 Practice

Solve each system by substitution.

1) $y = -3x$
$y = 6x - 9$

2) $y = x + 5$
$y = -2x - 4$

3) $y = -2x - 9$
$y = 2x - 1$

4) $y = -6x + 3$
$y = 6x + 3$

5) $y = 6x + 4$
$y = -3x - 5$

6) $y = 3x + 13$
$y = -2x - 22$

7) $y = 3x + 2$
$y = -3x + 8$

8) $y = -2x - 9$
$y = -5x - 21$

9) $y = 2x - 3$
$y = -2x + 9$

10) $y = 7x - 24$
$y = -3x + 16$

11) $y = 6x - 6$
$-3x - 3y = -24$

12) $-x + 3y = 12$
$y = 6x + 21$

13) $y = -6$
$3x - 6y = 30$

14) $6x - 4y = -8$
$y = -6x + 2$

15) $\begin{aligned} y &= -5 \\ 3x + 4y &= -17 \end{aligned}$

16) $\begin{aligned} 7x + 2y &= -7 \\ y &= 5x + 5 \end{aligned}$

17) $\begin{aligned} -2x + 2y &= 18 \\ y &= 7x + 15 \end{aligned}$

18) $\begin{aligned} y &= x + 4 \\ 3x - 4y &= -19 \end{aligned}$

19) $\begin{aligned} y &= -8x + 19 \\ -x + 6y &= 16 \end{aligned}$

20) $\begin{aligned} y &= -2x + 8 \\ -7x - 6y &= -8 \end{aligned}$

21) $\begin{aligned} 7x - 3y &= -7 \\ y &= 4x + 1 \end{aligned}$

22) $\begin{aligned} 3x - 2y &= -13 \\ 4x + 2y &= 18 \end{aligned}$

23) $\begin{aligned} x - 5y &= 7 \\ 2x + 7y &= -20 \end{aligned}$

24) $\begin{aligned} 3x - 4y &= 15 \\ 7x + y &= 4 \end{aligned}$

25) $\begin{aligned} -2x - 5y &= -5 \\ x - 8y &= -23 \end{aligned}$

26) $\begin{aligned} 6x + 4y &= 16 \\ -2x + 9y &= -3 \end{aligned}$

27) $\begin{aligned} -6x + y &= 20 \\ -3x - 3y &= -18 \end{aligned}$

28) $\begin{aligned} 7x + 5y &= -13 \\ x - 4y &= -16 \end{aligned}$

29) $\begin{aligned} 3x + y &= 9 \\ 2x + 8y &= -16 \end{aligned}$

30) $\begin{array}{l} -5x-5y=-20 \\ -2x+y=7 \end{array}$

31) $\begin{array}{l} 2x+y=2 \\ 3x+7y=14 \end{array}$

32) $\begin{array}{l} x=3y+4 \\ 2x-6y=8 \end{array}$

33) $\begin{array}{l} 4x-y=6 \\ y=4x-3 \end{array}$

34) $\begin{array}{l} y=x+7 \\ x-y=3 \end{array}$

35) $\begin{array}{l} -9x+6y=21 \\ 2y-7=3x \end{array}$

36) $\begin{array}{l} y=3x+5 \\ y=3x-1 \end{array}$

37) $\begin{array}{l} 3x-3y=-24 \\ x-y=5 \end{array}$

38) $\begin{array}{l} x+2y=1 \\ 5x=5-10y \end{array}$

39) $\begin{array}{l} 12x+6y=0 \\ y=-2x \end{array}$

40) $\begin{array}{l} -x-4y=-14 \\ -6x+8y=12 \end{array}$

2.2 Answers

1) $(1,-3)$
2) $(-3,2)$
3) $(-2,-5)$
4) $(0,3)$
5) $(-1,-2)$
6) $(-7,-8)$
7) $(1,5)$
8) $(-4,-1)$
9) $(3,3)$
10) $(4,4)$
11) $(2,6)$
12) $(-3,3)$
13) $(-2,-6)$
14) $(0,2)$
15) $(1,-5)$
16) $(-1,0)$
17) $(-1,8)$
18) $(3,7)$
19) $(2,3)$
20) $(8,-8)$
21) $\left(\frac{4}{5},\frac{21}{5}\right)$
22) $\left(\frac{5}{7},\frac{53}{7}\right)$
23) $(-3,-2)$
24) $(1,-3)$
25) $\left(-\frac{25}{7},\frac{17}{7}\right)$
26) $\left(\frac{78}{31},\frac{7}{31}\right)$
27) $(-2,8)$
28) $(-4,3)$
29) $(4,-3)$
30) $(-1,5)$
31) $(0,2)$
32) Infinite Solutions
33) No Solution

34) No Solution
35) Infinite Solutions
36) No Solution
37) No Solution
38) Infinite Solutions
39) Infinite Solutions
40) $(2, 3)$

Section 2.3: Solving Systems of Equations by Addition

Objective: Solve systems of equations using the addition method.

When solving systems we have found that graphing is very limited when solving equations. We then considered a second method known as substitution. This is probably the most used idea in solving systems in various areas of algebra. However, substitution can get ugly if we don't have a lone variable. This leads us to our third method for solving systems of equations: the **addition method** (sometimes called the elimination method). We will set up the process in the following examples, then define the five step process we can use to solve by addition.

Example 1.

Solve the systems of equations by addition: $\begin{matrix} 3x-4y=8 \\ 5x+4y=-24 \end{matrix}$

$3x-4y=8$ $\underline{5x+4y=-24}$	Notice opposites in front of y's. Add columns.
$8x\quad=-16$	
$\dfrac{8x}{8}=\dfrac{-16}{8}$	Solve for x, divide by 8
$x=-2$	We have our x!
$5(-2)+4y=-24$	Plug into either original equation, simplify
$-10+4y=-24$ $\underline{+10\qquad+10}$	Add 10 to both sides
$\dfrac{4y}{4}=\dfrac{-14}{4}$	Divide by 4
$y=-\dfrac{7}{2}$	Now we have our y!
$\left(-2,-\dfrac{7}{2}\right)$	Our Solution

In the previous example one variable had opposites in front of it, $-4y$ and $4y$. Adding these together eliminated the y completely. This allowed us to solve for the x. This is the idea behind the addition method. However, generally we won't have opposites in front of one of the variables. In this case we will manipulate the equations to get the opposites we want by multiplying one or both equations (on both sides!). This is shown in the next example.

Example 2.

Solve the systems of equations by addition: $\begin{aligned} -6x+5y&=22 \\ 2x+3y&=2 \end{aligned}$

$-6x+5y=22$ $2x+3y=2$	We can get opposites in front of x, by multiplying the second equation by 3, to get $-6x$ and $+6x$
$3(2+3y)=(2)3$	Distribute to get the new second equation.
$6x+9y=6$ $\underline{-6x+5y=22}$ $14y=28$	First equation still the same, add
$\dfrac{14y}{14}=\dfrac{28}{14}$	Divide both sides by 14
$y=2$	We have our y!
$2x+3(2)=2$	Plug into one of the original equations, simplify
$2x+6=2$ $\underline{\quad -6 \quad -6}$	Subtract 6 from both sides
$\dfrac{2x}{2}=\dfrac{-4}{2}$	Divide both sides by 2
$x=-2$	We also have our x!
$(-2, 2)$	Our Solution

When we looked at the x terms, $-6x$ and $2x$ we decided to multiply the $2x$ by 3 to get the opposites we were looking for. What we are looking for with our opposites is the least common multiple (LCM) of the coefficients. We also could have solved the above problem by looking at the terms with y, $5y$ and $3y$. The LCM of 3 and 5 is 15. So we would want to multiply both equations, the $5y$ by 3, and the $3y$ by -5 to get opposites, $15y$ and $-15y$.

This illustrates an important point: for some problems we will have to multiply both equations by a constant (on both sides) to get the opposites we want before adding.

Example 3.

Solve the systems of equations by addition: $\begin{aligned} 3x+6y&=-9 \\ 2x+9y&=-26 \end{aligned}$

$3x+6y=-9$	We can get opposites in front of y, find LCM of 6 and 9
$2x+9y=-26$	The LCM is 18. We will multiply to get $18y$ and $-18y$
$3(3x+6y)=(-9)3$ $9x+18y=-27$	Multiply the first equation by 3, both sides!

$-2(2x+9y)=(-26)(-2)$	Multiply the second equation by -2, both sides!
$-4x-18y=52$	
$9x+18y=-27$ $-4x-18y=52$	Add the two new equations together
$5x \quad = 25$	Divide both sides by 5
$\frac{5x}{5}=\frac{25}{5}$	
$x=5$	We have our solution for x
$3(5)+6y=-9$	Plug into either original equation; simplify
$15+6y=-9$ $-15 \quad -15$	Subtract 15 from both sides
$\frac{6y}{6}=\frac{-24}{6}$	Divide both sides by 6
$y=-4$	Now we have our solution for y
$(5,-4)$	Our Solution

It is important for each problem as we get started that all variables and constants are lined up before we start multiplying and adding equations. This is illustrated in the next example, which includes the five steps we will go through to solve a problem using addition.

Problem	$2x-5y=-13$ $-3y+4=-5x$
1. Line up the variables and constants	Second Equation: $-3y+4=-5x$ $+5x-4 \quad +5x-4$ $5x-3y=-4$
2. Multiply to get opposites (use LCM)	$2x-5y=-13$ $5x-3y=-4$ First Equation: multiply by -5 $-5(2x-5y)=(-13)(-5)$ $-10x+25y=65$ Second Equation: multiply by 2 $2(5x-3y)=(-4)2$ $10x-6y=-8$ $-10x+25y=65$ $10x-6y=-8$

3. Add	$19y = 57$
4. Solve	$\frac{19y}{19} = \frac{57}{19}$ $y = 3$
5. Plug into either original and solve	$2x - 5(3) = -13$ $2x - 15 = -13$ $\quad +15 \quad +15$ $\frac{2x}{2} = \frac{2}{2}$ $x = 1$
Solution	$(1,3)$

World View Note: The famous mathematical text, *The Nine Chapters on the Mathematical Art*, which was printed around 179 AD in China describes a formula very similar to Gaussian elimination which is very similar to the addition method.

Just as with graphing and substitution, it is possible to have no solution or infinite solutions with addition. Just as with substitution, if the variables all disappear from our problem, a true statement will indicate infinite solutions and a false statement will indicate no solution.

Example 4.

Solve the systems of equations by addition: $\begin{aligned} 2x - 5y &= 3 \\ -6x + 15y &= -9 \end{aligned}$

$$\begin{aligned} 2x - 5y &= 3 \\ -6x + 15y &= -9 \end{aligned}$$ To get opposites in front of x, multiply first equation by 3

$$\begin{aligned} 3(2x - 5y) &= (3)3 \\ 6x - 15y &= 9 \end{aligned}$$ Distribute

$$\begin{aligned} 6x - 15y &= 9 \\ \underline{-6x + 15y} &\underline{= -9} \end{aligned}$$ Add equations together

$0 = 0$ True statement

Infinite solutions Our Solution

Example 5.

Solve the systems of equations by addition: $\begin{array}{l} 4x-6y=8 \\ 6x-9y=15 \end{array}$

$$\begin{array}{rl}
4x-6y=8 & \text{LCM for } x\text{'s is 12} \\
6x-9y=15 & \\
& \\
3(4x-6y)=(8)3 & \text{Multiply first equation by 3} \\
12x-18y=24 & \\
& \\
-2(6x-9y)=(15)(-2) & \text{Multiply second equation by } -2 \\
-12x+18y=-30 & \\
& \\
12x-18y=24 & \text{Add both new equations together} \\
\underline{-12x+18y=-30} & \\
0=-6 & \text{False statement} \\
\text{No Solution} & \text{Our Solution}
\end{array}$$

We have covered three different methods that can be used to solve a system of two equations with two variables. While all three can be used to solve any system, graphing works well for small integer solutions. Substitution works well when we have a lone variable, and addition works well when the other two methods fail. As each method has its own strengths, it is important you are familiar with all three methods.

2.3 Practice

Solve each system by the addition (or elimination) method.

1) $\begin{array}{l} 4x+2y=0 \\ -4x-9y=-28 \end{array}$

2) $\begin{array}{l} -7x+y=-10 \\ -9x-y=-22 \end{array}$

3) $\begin{array}{l} -9x+5y=-22 \\ 9x-5y=13 \end{array}$

4) $\begin{array}{l} -x-2y=-7 \\ x+2y=7 \end{array}$

5) $\begin{array}{l} -6x+9y=3 \\ 6x-9y=-9 \end{array}$

6) $\begin{array}{l} 5x-5y=15 \\ -10x+10y=-30 \end{array}$

7) $\begin{array}{l} 4x-6y=-10 \\ 4x-6y=-14 \end{array}$

8) $\begin{array}{l} -3x+3y=-12 \\ -3x+9y=-24 \end{array}$

9) $\begin{array}{l} -x-5y=28 \\ -x+4y=-17 \end{array}$

10) $\begin{array}{l} -10x-5y=0 \\ -10x-10y=-30 \end{array}$

11) $\begin{array}{l} 2x-y=5 \\ 5x+2y=-28 \end{array}$

12) $\begin{array}{l} -5x+6y=-17 \\ x-2y=5 \end{array}$

13) $\begin{array}{l} 10x+6y=24 \\ -6x+y=4 \end{array}$

14) $\begin{array}{l} x+3y=-12 \\ 10x+6y=-10 \end{array}$

15) $\begin{aligned} 2x+4y&=25 \\ 4x-12y&=7 \end{aligned}$

16) $\begin{aligned} -6x+4y&=12 \\ 12x+6y&=18 \end{aligned}$

17) $\begin{aligned} -7x+4y&=-4 \\ 10x-8y&=-8 \end{aligned}$

18) $\begin{aligned} -6x+4y&=4 \\ -3x-y&=26 \end{aligned}$

19) $\begin{aligned} 5x+10y&=20 \\ -6x-5y&=-3 \end{aligned}$

20) $\begin{aligned} -9x-5y&=-19 \\ 3x-7y&=-11 \end{aligned}$

21) $\begin{aligned} -7x-3y&=12 \\ -6x-5y&=20 \end{aligned}$

22) $\begin{aligned} -5x+4y&=4 \\ -7x-10y&=-10 \end{aligned}$

23) $\begin{aligned} 9x-2y&=-18 \\ 5x-7y&=-10 \end{aligned}$

24) $\begin{aligned} 3x+7y&=-8 \\ 4x+6y&=-4 \end{aligned}$

25) $\begin{aligned} 9x+6y&=-21 \\ -10x-9y&=28 \end{aligned}$

26) $\begin{aligned} -4x-5y&=12 \\ -10x+6y&=30 \end{aligned}$

27) $\begin{aligned} -7x+5y&=-18 \\ -3x-3y&=12 \end{aligned}$

28) $\begin{aligned} 8x+7y&=-24 \\ 6x+3y&=-18 \end{aligned}$

29) $\begin{aligned} -8x-8y&=-8 \\ 10x+9y&=1 \end{aligned}$

30) $\begin{matrix} -7x+10y=13 \\ 4x+9y=22 \end{matrix}$

31) $\begin{matrix} 9y=7-x \\ -18y+4x=-26 \end{matrix}$

32) $\begin{matrix} 2x+5y=-9 \\ 6x-10y=3 \end{matrix}$

2.3 Answers

1) $(-2, 4)$
2) $(2, 4)$
3) No solution
4) Infinite number of solutions
5) No solution
6) Infinite number of solutions
7) No solution
8) $(2, -2)$
9) $(-3, -5)$
10) $(-3, 6)$
11) } $(-2, -9)$
12) $(1, -2)$
13) $(0, 4)$
14) $\left(\frac{7}{4}, -\frac{55}{12}\right)$
15) $\left(\frac{41}{5}, \frac{43}{20}\right)$
16) $(0, 3)$
17) $(4, 6)$
18) $(-6, -8)$
19) $(-2, 3)$
20) $(1, 2)$
21) $(0, -4)$
22) $(0, 1)$
23) $(-2, 0)$
24) $(2, -2)$
25) $(-1, -2)$
26) $(-3, 0)$
27) $\left(-\frac{1}{6}, -\frac{23}{6}\right)$
28) $(-3, 0)$
29) $(-8, 9)$
30) $(1, 2)$
31) $(-2, 1)$
32) $\left(-\frac{3}{2}, -\frac{6}{5}\right)$

Section 2.4: Applications of Systems

Objective: Solve application problems by setting up a system of equations.

One application of system of equations are known as value problems. Value problems are ones where each variable has a value attached to it. For example, if our variable is the number of nickels in a person's pocket, those nickels would have a value of five cents each. We will use a table to help us set up and solve value problems. The basic structure of the table is shown below.

	Number	Value	Total
Item 1			
Item 2			
Total			

The first column in the table is used for the number of things we have. Quite often these will be our variables. The second column is used for the value each item has. The third column is used for the total value which we calculate by multiplying the number by the value. For example, if we have 7 dimes, each with a value of 10 cents, the total value is $(7)(10)=70$ cents. The last row of the table is for totals. We only will use the third row (also marked total) for the totals that are given to use. This means sometimes this row may have some blanks in it.

Once the table is filled in we can easily make equations by adding each column, setting it equal to the total at the bottom of the column. This is shown in the following example.

Example 1.

In a child's bank there are 11 coins that have a value of \$1.85. The coins are either quarters or dimes. How many coins of each type does the child have?

	Number	Value	Total
Quarter	q	25	
Dime	d	10	
Total			

Using value table; use q for quarters. d for dimes
Each quarter's value is 25 cents, each dime's is 10 cents

	Number	Value	Total
Quarter	q	25	$25q$
Dime	d	10	$10d$
Total			

Multiply number by value to get totals

	Number	Value	Total
Quarter	q	25	$25q$
Dime	d	10	$10d$
Total	11		185

We have 11 coins total. This is the number total.
We have 1.85 for the final total,
Write final total in cents (185) because 25 and 10 are written using cents

$$q + d = 11$$
$$25q + 10d = 185$$

First and last columns are our equations by adding
Solve by either addition or substitution.
Here we will use the addition method.

$$-10(q+d) = (11)(-10)$$ Multiply first equation by -10
$$-10q - 10d = -110$$

$$-10q - 10d = -110$$ Add together equations
$$25q + 10d = 185$$
$$15q = 75$$

$$\frac{15q}{15} = \frac{75}{15}$$ Divide both sides by 15

$$q = 5$$ We have our q, number of quarters is 5

$$(5) + d = 11$$ Plug into one of original equations
$$-5 \quad -5$$ Subtract 5 from both sides
$$d = 6$$ We have our d, number of dimes is 6

5 quarters and 6 dimes — Our Solution

World View Note: American coins are the only coins that do not state the value of the coin in numeric form. On the back of the dime it says "one dime" (not 10 cents). On the back of the quarter it says "one quarter" (not 25 cents). On the penny it says "one cent" (not 1 cent). The rest of the world (Euros, Yen, Pesos, etc) all write the value as a number so people who don't speak the language can easily use the coins.

Ticket sales also have a value. Often different types of tickets sell for different prices (values). These problems can be solved in much the same way.

Example 2.
There were 41 tickets sold for an event. Tickets for children cost $1.50 and tickets for adults cost $2.00. Total receipts for the event were $73.50. How many of each type of ticket were sold?

	Number	Value	Total
Child	c	1.5	
Adult	a	2	
Total			

Using our value table, c for child, a for adult
Child tickets have value 1.50; adult value is 2.00
(we can drop the zeros after the decimal point)

	Number	Value	Total
Child	c	1.5	$1.5c$
Adult	a	2	$2a$
Total			

Multiply number by value to get totals

	Number	Value	Total
Child	c	1.5	$1.5c$
Adult	a	2	$2a$
Total	41		73.5

We have 41 tickets sold. This is our number total
The final total was 73.50
Write in dollars as 1.5 and 2 are also dollars

$$c + a = 41$$
$$1.5c + 2a = 73.5$$

First and last columns are our equations by adding
We can solve by either addition or substitution

$$c + a = 41$$ We will solve by substitution.

$$-c \qquad -c$$ Solve for a by subtracting c

$$a = 41 - c$$

$$1.5c + 2(41 - c) = 73.5$$ Substitute into untouched equation

$$1.5c + 82 - 2c = 73.5$$ Distribute

$$-0.5c + 82 = 73.5$$ Combine like terms

$$-82 \quad -82$$ Subtract 82 from both sides

$$\frac{-0.5c}{-0.5} = \frac{-8.5}{-0.5}$$ Divide both sides by -0.5

$$c = 17$$ We have c, number of child tickets is 17

$$a = 41 - (17)$$ Plug into $a =$ equation to find a

$$a = 24$$ We have our a, number of adult tickets is 24

17 child tickets and 24 adult tickets — Our Solution

Some problems will not give us the total number of items we have. Instead they will give a relationship between the items. Here we will have statements such as "There are twice as many dimes as nickels". While it is clear that we need to multiply one variable by 2, it may not be clear which variable gets multiplied by 2.
Generally the equations are backwards from the English sentence. If there are twice as many nickels as dimes, than we multiply the other variable (dimes) by two. So, the equation would be $n = 2d$. This type of problem is in the next example.

Example 3.

A man has a collection of stamps made up of 5¢ stamps and 8¢ stamps. There are three times as many 8¢ stamps as 5¢ stamps. The total value of all the stamps is $3.48. How many of each stamp does he have?

	Number	Value	Total
Five	f	5	
Eight	e	8	
Total			

Use value table, f for five cent stamp, and e for eight
Also list value of each stamp under value column

	Number	Value	Total
Five	f	5	$5f$
Eight	e	8	$8e$
Total			

Multiply number by value to get total

	Number	Value	Total
Five	f	5	$5f$
Eight	e	8	$8e$
Total			348

The final total was 348 (written in cents)
We do not know the total number, this is left blank.

$e = 3f$ **3 times as many 8¢ stamps as 5¢ stamps**
$5f + 8e = 348$ Total column gives second equation

$5f + 8(3f) = 348$ Substitution, substitute first equation in second
$5f + 24f = 348$ Multiply first
$29f = 348$ Combine like terms
$\frac{29f}{29} = \frac{348}{29}$ Divide both sides by 39
$f = 12$ We have f. There are 12 five cent stamps
$e = 3(12)$ Plug into first equation
$e = 36$ We have e, There are 36 eight cent stamps
12 five cent, 36 eight cent stamps Our Solution

The same process for solving value problems can be applied to solving interest problems. Our table titles will be adjusted slightly as we do so.

	Invest	Rate	Interest
Account 1			
Account 2			
Total			

Our first column is for the amount invested in each account. The second column is the interest rate earned (written as a decimal - move decimal point twice left), and the last column is for the amount of interest earned. Just as before, we multiply the investment amount by the rate to find the final column, the interest earned. This is shown in the following example.

Example 4.

A woman invests \$4000 in two accounts, one at 6% interest, the other at 9% interest for one year. At the end of the year she had earned \$270 in interest. How much did she have invested in each account?

	Invest	Rate	Interest
Account 1	x	0.06	
Account 2	y	0.09	
Total			

Use our investment table, x and y for accounts
Fill in interest rates as decimals
Note: 6% = 0.06 and 9% = 0.09

	Invest	Rate	Interest
Account 1	x	0.06	$0.06x$
Account 2	y	0.09	$0.09y$
Total			

Multiply across to find interest earned.

	Invest	Rate	Interest
Account 1	x	0.06	$0.06x$
Account 2	y	0.09	$0.09y$
Total	4000		270

Total investment is 4000,
Total interest was 270

$$x + y = 4000$$
$$0.06x + 0.09y = 270$$

First and last column give our two equations
Solve by either substitution or addition

$$(-0.06)(x + y) = (4000)(-0.06)$$
$$-0.06x - 0.06y = -240$$

Using addition, multiply the first equation by -0.06

$$\begin{array}{r} -0.06x - 0.06y = -240 \\ 0.06x + 0.09y = 270 \\ \hline \end{array}$$

Add equations together

$$\frac{0.03y}{0.03} = \frac{30}{0.03}$$

Divide both sides by 0.03

$$y = 1000$$

We have y = \$1000 invested at 9%

$$\begin{array}{r} x + 1000 = 4000 \\ -1000 \quad -1000 \\ \hline x = 3000 \end{array}$$

Plug into original equation
Subtract 1000 from both sides
We have x = \$3000 invested at 6%

\$1000 at 9% and \$3000 at 6% Our Solution

The same process can be used to find an unknown interest rate.

Example 5.

John invests \$5000 in one account and \$8000 in an account paying 4% more in interest. He earned \$1230 in interest after one year. At what rates did he invest?

	Invest	Rate	Interest
Account 1	5000	x	
Account 2	8000	$x + 0.04$	
Total			

Use our investment table. Use x for first rate
The second rate is 4% higher, or $x + 0.04$
Be sure to write this rate as a decimal!

	Invest	Rate	Interest
Account 1	5000	x	$5000x$
Account 2	8000	$x+0.04$	$8000(x+0.04)$
Total			

Multiply to fill in interest column.
Be sure to distribute $8000(x+0.04)$

	Invest	Rate	Interest
Account 1	5000	x	$5000x$
Account 2	8000	$x+0.04$	$8000x+320$
Total			1230

Total interest was 1230.

$5000x+8000x+320=1230$ Last column gives our equation

$13000x+320=1230$ Combine like terms

$-320 \quad -320$ Subtract 320 from both sides

$\frac{13000x}{13000}=\frac{910}{13000}$ Divide both sides by 13000

$x=0.07$ We have our x, 0.07 or 7% interest

$(0.07)+0.04$ Second account is 4% higher

0.11 The account with \$8000 is at 0.11 or 11% interest

\$5000 at 7%, and \$8000 at 11% Our Solution

2.4 Practice

Solve.

1) A collection of dimes and quarters is worth $15.25. There are 103 coins in all. How many of each are there?

2) A collection of half dollars and nickels is worth $13.40. There are 34 coins in all. How many of each are there?

3) The attendance at a school concert was 578. Admission was $2.00 for adults and $1.50 for children. The total receipts were $985.00. How many adults and how many children attended?

4) A purse contains $3.90 made up of dimes and quarters. If there are 21 coins in all, how many dimes and how many quarters are there?

5) A boy has $2.25 in nickels and dimes. If there are twice as many dimes as nickels, how many of each kind does he have?

6) $3.75 is made up of quarters and half dollars. If the number of quarters exceeds the number of half dollars by 3, how many coins of each denomination are there?

7) A collection of 27 coins consisting of nickels and dimes amounts to $2.25. How many coins of each kind are there?

8) $3.25 in dimes and nickels were distributed among 45 boys. If each received one coin, how many received a dime and how many received a nickel?

9) There were 429 people at a play. Admission was $1 each for adults and 75 cents each for children. The receipts were $372.50. How many children and how many adults attended?

10) There were 200 tickets sold for a women's basketball game. Tickets for students were 50 cents each and for adults 75 cents each. The total amount of money collected was $132.50. How many of each type of ticket was sold?

11) There were 203 tickets sold for a volleyball game. For activity-card holders, the price was $1.25 each and for non-card holders the price was S2 each. The total amount of money collected was $310. How many of each type of ticket was sold?

12) At a local ball game the hotdogs sold for $2.50 each and the hamburgers sold for $2.75 each. There were 131 total sandwiches sold for a total value of $342. How many of each sandwich was sold?

13) At a recent Vikings game $445 in admission tickets was taken in. The cost of a student ticket was $1.50 and the cost of a non-student ticket was $2.50. A total of 232 tickets were sold. How many students and how many nonstudents attended the game?

14) A bank contains 27 coins in dimes and quarters. The coins have a total value of $4.95. Find the number of dimes and quarters in the bank.

15) A coin purse contains 18 coins in nickels and dimes. The coins have a total value of $1.15. Find the number of nickels and dimes in the coin purse.

16) A business executive bought 40 stamps for $9.60. The purchase included 25¢ stamps and 20¢ stamps. How many of each type of stamp were bought?

17) A postal clerk sold some 15¢ stamps and some 25¢ stamps. Altogether, 15 stamps were sold for a total cost of $3.15. How many of each type of stamp were sold?

18) A drawer contains 15¢ stamps and 18¢ stamps. The number of 15¢ stamps is four less than three times the number of 18¢ stamps. The total value of all the stamps is $1.29. How many 15¢ stamps are in the drawer?

19) The total value of dimes and quarters in a bank is $6.05. There are six more quarters than dimes. Find the number of each type of coin in the bank.

20) A child's piggy bank contains 44 coins in quarters and dimes. The coins have a total value of $8.60. Find the number of quarters in the bank.

21) A coin bank contains nickels and dimes. The number of dimes is 10 less than twice the number of nickels. The total value of all the coins is $2.75. Find the number of each type of coin in the bank.

22) A total of 26 bills are in a cash box. Some of the bills are one dollar bills, and the rest are five dollar bills. The total amount of cash in the box is $50. Find the number of each type of bill in the cash box.

23) A bank teller cashed a check for $200 using twenty dollar bills and ten dollar bills. In all, twelve bills were handed to the customer. Find the number of twenty dollar bills and the number of ten dollar bills.

24) A collection of stamps consists of 22¢ stamps and 40¢ stamps. The number of 22¢ stamps is three more than four times the number of 40¢ stamps. The total value of the stamps is $8.34. Find the number of 22¢ stamps in the collection.

25) A total of $27000 is invested, part of it at 12% and the rest at 13%. The total interest after one year is $3385. How much was invested at each rate?

26) A total of $50000 is invested, part of it at 5% and the rest at 7.5%. The total interest after one year is $3250. How much was invested at each rate?

27) A total of $9000 is invested, part of it at 10% and the rest at 12%. The total interest after one year is $1030. How much was invested at each rate?

28) A total of $18000 is invested, part of it at 6% and the rest at 9%. The total interest after one year is $1248. How much was invested at each rate?

29) An inheritance of $10000 is invested in 2 ways, part at 9.5% and the remainder at 11%. The combined annual interest was $1038.50. How much was invested at each rate?

30) Kerry earned a total of $900 last year on his investments. If $7000 was invested at a certain rate of return and $9000 was invested in a fund with a rate that was 2% higher, find the two rates of interest.

31) Jason earned $256 interest last year on his investments. If $1600 was invested at a certain rate of return and $2400 was invested in a fund with a rate that was double the rate of the first fund, find the two rates of interest.

32) Millicent earned $435 last year in interest. If $3000 was invested at a certain rate of return and $4500 was invested in a fund with a rate that was 2% lower, find the two rates of interest.

33) A total of $8500 is invested, part of it at 6% and the rest at 3.5%. The total interest after one year is $385. How much was invested at each rate?

34) A total of $12000 was invested, part of it at 9% and the rest at 7.5%. The total interest after one year is $1005. How much was invested at each rate?

35) A total of $15000 is invested, part of it at 8% and the rest at 11%. The total interest after one year is $1455. How much was invested at each rate?

36) A total of $17500 is invested, part of it at 7.25% and the rest at 6.5%. The total interest after one year is $1227.50. How much was invested at each rate?

37) A total of $6000 is invested, part of it at 4.25% and the rest at 5.75%. The total interest after one year is $300. How much was invested at each rate?

38) A total of $14000 is invested, part of it at 5.5% and the rest at 9%. The total interest after one year is $910. How much was invested at each rate?

39) A total of $11000 is invested, part of it at 6.8% and the rest at 8.2%. The total interest after one year is $797. How much was invested at each rate?

40) An investment portfolio earned \$2010 in interest last year. If \$3000 was invested at a certain rate of return and \$24000 was invested in a fund with a rate that was 4% lower, find the two rates of interest.

41) Samantha earned \$1480 in interest last year on her investments. If \$5000 was invested at a certain rate of return and \$11000 was invested in a fund with a rate that was two-thirds the rate of the first fund, find the two rates of interest.

2.4 Answers

1) 33 quarters, 70 dimes
2) 26 half dollars, 8 nickels
3) 236 adult, 342 child
4) 9 dimes, 12 quarters
5) 9 nickels, 18 dimes
6) 7 quarters, 4 half dollars
7) 9 nickels, 18 dimes
8) 25 nickels, 20 dimes
9) 203 adults, 226 child
10) 130 adults, 70 students
11) 128 card, 75 non-card
12) 73 hot dogs, 58 hamburgers
13) 135 students, 97 non-students
14) 12 dimes, 15 quarters
15) 13 nickels, 5 dimes
16) 8 20¢ stamps, 32 25¢ stamps
17) 6 15¢ stamps, 9 25¢ stamps
18) 5 15¢ stamps
19) 13 dimes, 19 quarters
20) 28 quarters
21) 15 nickels, 20 dimes
22) 20 \$1 bills, 6 \$5 bills
23) 8 \$20 bills, 4 \$10 bills
24) 27 22¢ stamps
25) \$12500 @ 12% , \$14500 @ 13%
26) \$20000 @ 5%, \$30000 @ 7.5%
27) \$2500 @ 10%, \$6500 @ 12%
28) \$12400 @ 6%, \$5600 @ 9%
29) \$4100 @ 9.5%, \$5900 @ 11%
30) \$7000 @ 4.5%, \$9000 @ 6.5%
31) \$1600 @ 4%; \$2400 @ 8%
32) \$3000 @ 7%, \$4500 @ 5%
33) \$3500 @ 6%; \$5000 @ 3.5%
34) \$7000 @ 9%, \$5000 @ 7.5%
35) \$6500 @ 8%; \$8500 @ 11%
36) \$12000 @ 7.25%, \$5500 @ 6.5%
37) \$3000 @ 4.25%; \$3000 @ 5.75%
38) \$10000 @ 5.5%, \$4000 @ 9%
39) \$7500 @ 6.8%; \$3500 @ 8.2%
40) \$3000 @ 11%; \$24000 @ 7%
41) \$5000 @ 12%, \$11000 @ 8%