4.1 Graphing Linear Equations and Inequalities

A **linear equation in two variables** is one that can be written in the form $ax + by = c$ where a, b, and c are real numbers but $a \neq 0$ and $b \neq 0$. There are several methods that can be used to graph linear equations in two variables. Two methods will be reviewed: plotting points and graphing by intercepts.

4.1.1 Graphing by Plotting Points

The **plotting points method** can be used in any case. You choose random numbers for one variable and then solve for the other. Each time this is complete, you have found a point along the graph of the line. You repeat this at least once to get a second point. Finding a third point serves as a check. The example below uses three points. If the three points do not form a straight line, return and check your work.

Example 4.1.1

Graph $2x + y = 8$ by plotting points.

<table>
<tr><th>Procedure
(in words)</th><th>Procedure
(in action)</th><th>Graph</th></tr>
<tr>
<td>Pick any random value for x or y. Substitute this value into the equation and solve for the remaining variable. You now have one point that is a part of the graph. Repeat for more points (at least two total).</td>
<td>
<table>
<tr><th>x</th><th>y</th><th>(x, y)</th></tr>
<tr><td>0</td><td>$2(0) + y = 8$
$0 + y = 8$
$y = 8$</td><td>(0,8)</td></tr>
<tr><td>1</td><td>$2(1) + y = 8$
$2 + y = 8$
$y = 6$</td><td>(1,6)</td></tr>
<tr><td>2</td><td>$2(2) + y = 8$
$4 + y = 8$
$y = 4$</td><td>(2,4)</td></tr>
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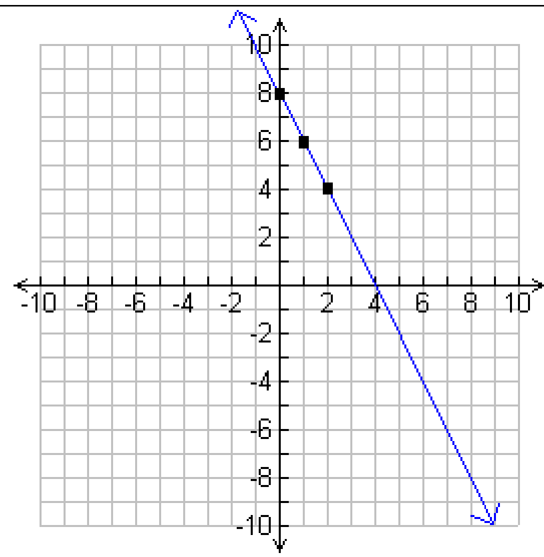
</td>
<td>
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</table>

The plotting points method can be used at any given time, but it is sometimes more tedious than other methods. This method isn't particularly useful when graphing curves, although those graphs are not the focus of this course.

Try it Now 4.1.1

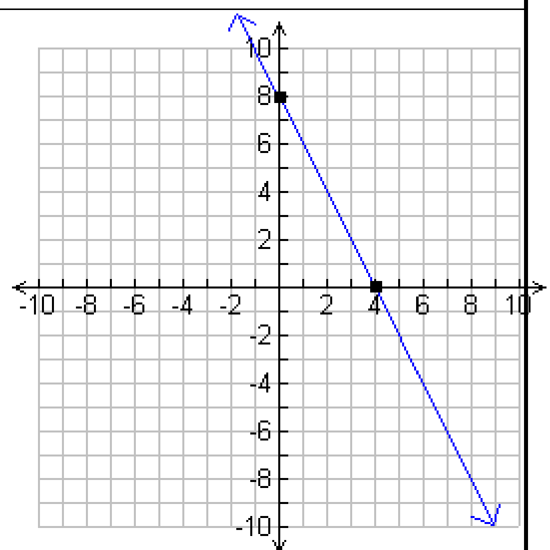
Graph $y = 5x - 3$ by plotting points.

4.1.2 Graphing Using Intercepts

The next method discussed is the **intercept method**. Recall that the *x*-intercept of the graph of an equation is the point(s) where the graph intersects the *x*-axis. Similarly, the *y*-intercept of the graph of an equation is the point(s) where the graph intersects the *y*-axis. This method is mostly convenient for graphing lines, but for graphs of other equations that we won't examine in this course (such as circles), these intercepts sometimes don't exist. Let's review graphing the equation of a line using the intercept method.

Example 4.1.2

Graph $2x + y = 8$ using the intercept method.

Procedure (in words)	**Procedure (in action)**	**Graph**
To find the *x*-intercept, the point where the graph intercepts the *x* axis, let $y = 0$. Solve for x. You now have a coordinate pair representing the *x*-intercept. To find the *y*-intercept, the point where the graph intercepts the *y* axis, let $x = 0$. Solve for y. You now have a coordinate pair representing the *y*-intercept.	<u>*x*-intercept</u> $2x + 0 = 8$ $2x = 8$ $x = 4$ <u>*y*-intercept</u> $2(0) + y = 8$ $0 + y = 8$ $y = 8$ (0,8)	

Notice the equation used on Example 1 and Example 2 were the same. We used two different graphing techniques and both resulted in the same graph.

Try it Now 4.1.2

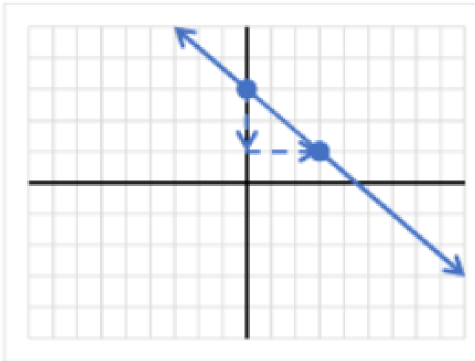
Graph $-3x + 12y = 12$ using the intercept method.

4.1.3 Graphing Using Slope-Intercept

The next method discussed is the **slope-intercept method**. Recall that the slope of a line can be thought of as $m = \frac{rise}{run} = \frac{\Delta y}{\Delta x}$. This method is best used for lines of the form $y = mx + b$, where b represents the y-intercept. Let's review graphing the equation of a line using the slope-intercept method.

Example 4.1.3

Graph $y = -\frac{2}{3}x + 3$ using the slope-intercept method.

Procedure (in words)	Procedure (in action)	Graph
The y-intercept is given by b in the equation $y = mx + b$ We think of the slope, m, as the fraction $\frac{change\ in\ y}{change\ in\ x}$, where the change in y represents how far to go up/down (depending on sign) the y-axis from the y-intercept, and the change in x tells us how far to go right/left (depending on sign).	<u>y-intercept</u> $b = 3$ <u>slope</u> $m = -\frac{2}{3}$ change in y=-2 change in x=3	

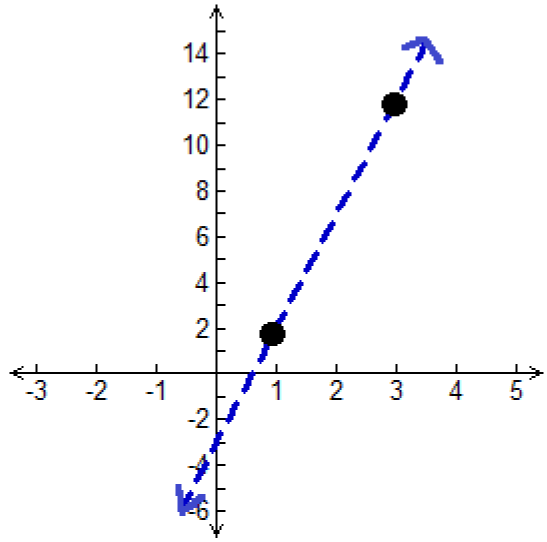
The method you decide to use, in upcoming sections, will be up to you.

An **inequality** is a statement containing one of these symbols: $<, \leq, >, \geq$. When we graph a linear inequality, we have an infinite solution set. For example, consider the inequality $x + y > 9$. One possible solution is (10,12) since 10+12>9. Another solution is (-8,20) since -8+20>9. We could list ordered pairs that are solutions to this inequality, for the rest of our lives! After all, $(1000000{,}300000000)$ is a solution too! When we graph an inequality, we are constructing a visual representation of all possible solutions. We will examine a few examples.

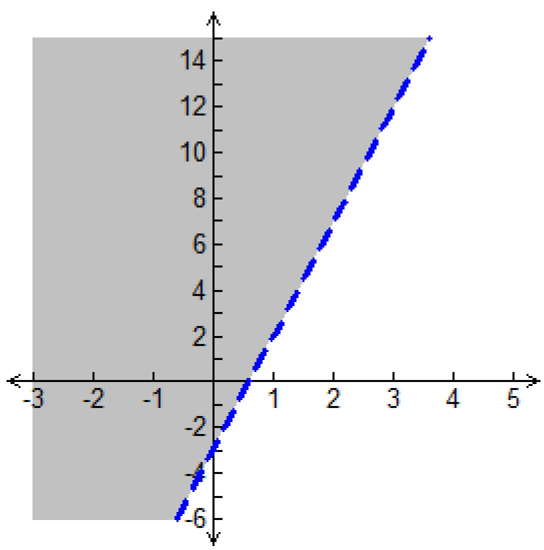
Example 4.1.4

Graph $y > 5x - 3$.

We first graph the equation $y = 5x - 3$. Let's use the slope-intercept method. The y-intercept will be $(0, -3)$. From there we will use a slope of $m = 5 = \frac{5}{1}$ to generate a second point on the graph by "rising" 5 up, and "running" 1 to the right.
Typically, we draw a solid line. However, inequalities are different. Points along this line don't actually satisfy the original inequality. For example, consider the point (3,12). When you substitute these values into the original inequality, you get 12>5(3)-3 or 12>12. This is false. Therefore, we will use a dotted line when connecting the points. Whenever you have an inequality with a < or > symbol, a dashed line will always be used. Solid lines with be used with ≤ and ≥ symbols and we'll examine this in the next example.



We're not quite finished with the graph. We have to determine which side of the line represents the solution set. To determine which side of the line, test any point not on the line by substituting its coordinates for *x* and *y* in the inequality. Suppose we test the point (0,0) because it is easy to work with: 0>5(0)-3. This results in 0>-3, which is a true statement. This means that every point on the same side of the line as (0,0) satisfies the inequality. (Try a few if you don't believe it. Every point on the other side of the line makes the inequality false.) We want to shade the side of the line that represents the solution set. Therefore, we shade the northwestern part of the graph.



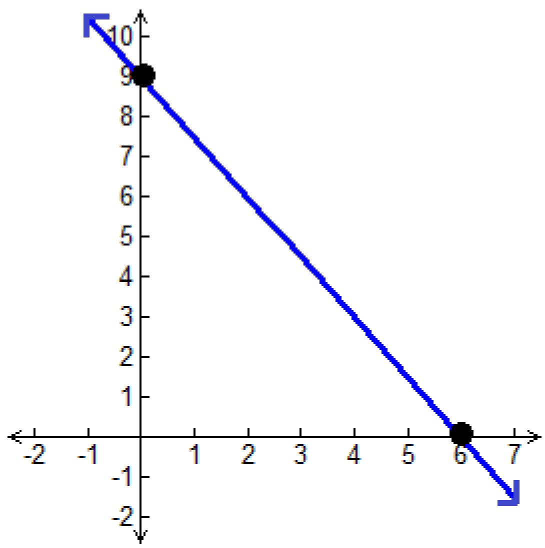
Example 4.1.4

Graph $3x + 2y \geq 18$using the intercept method.

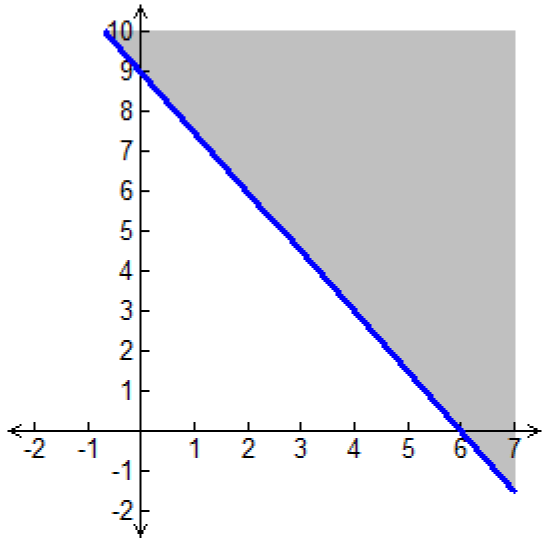
We first graph the equation $3x + 2y = 18$. For variety, let's graph this using the intercept method (although the plotting points method will work too).

x	y	(x, y)
0	$3(0) + 2y = 18$ $2y = 18$ $y = 9$	(0,9)
$3x + 2(0) = 18$ $3x = 18$ $x = 6$	0	(6,0)

Plot the points on a graph. In the previous example, we connected these with a dashed line. This inequality contains a ≤ symbol, meaning the points we determined (the intercepts) actually satisfy the original inequality. Try one: $3(0) + 2(9) \geq 18$ gives $18 \geq 18$, which is a true statement. Therefore, a solid line will be used. Remember, whenever you have an inequality with a < or > symbol, a dashed line will always be used and whenever you have an inequality with a ≤ or ≥ symbol, a solid line will be used.



Now we are finished graphing the equation (the line). Return to the inequality $3x + 2y \geq 18$. To determine which side of the line, test any point not on the line by substituting its coordinates for *x* and *y* in the inequality. Suppose we test the point (0,0) because it is easy to work with: $3(0) + 2(0) \geq 18$. This results in $0 \geq 18$, which is a false statement. This means that every point on the same side of the line as $(0,0)$ *does not* satisfy the inequality. (Try a few if you don't believe it. Every point on the other side of the line makes the inequality true.) We want to shade the side of the line that does represent the solution set. Therefore, we shade the northeastern part of the graph.



Try it Now 4.1.3

Graph $4x + 12y \leq 24$.

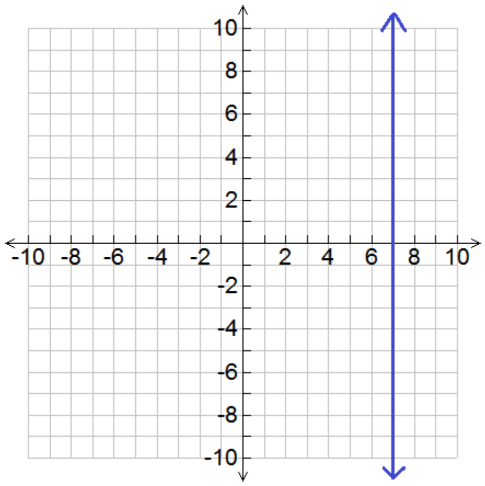
4.1.4 Graphing Horizontal and Vertical Lines

At the beginning of this section, we defined a linear equation in two variables is one that can be written in the form $ax + by = c$ where a, b, and c are real numbers but $a \neq 0$ and $b \neq 0$. If a or b is zero, we have a linear equation in one variable. Graphs of these equations are still linear but provide horizontal or vertical lines.

Example 4.1.5

Graph $x = 7$.

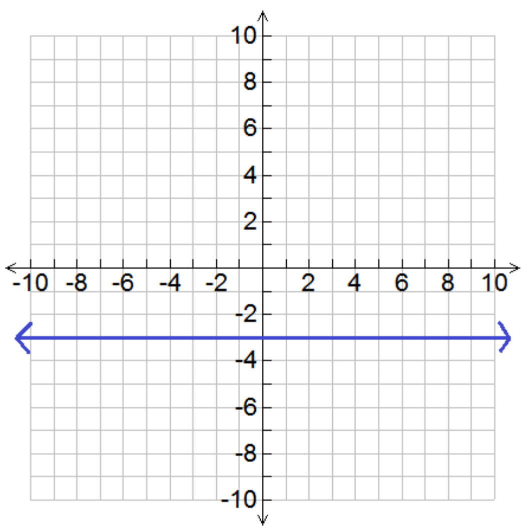
The graph of this equation represents the set of points where $x = 7$. Ordered pairs where $x = 7$include $(7, -4)$, $(7,0)$, or $(7,6)$. The y values can include any real number. If you plot these points, you will notice a vertical line graph forming:



Example 4.1.6

Graph $y = -3$.

The graph of this equation represents the set of points where $y = -3$. Ordered pairs where $y = -3$.include $(-4, -3)$, $(0, -3)$, or $(5, -3)$. The x values can include any real number. If you plot these points, you will notice a horizontal line graph forming:



Try it Now Answers

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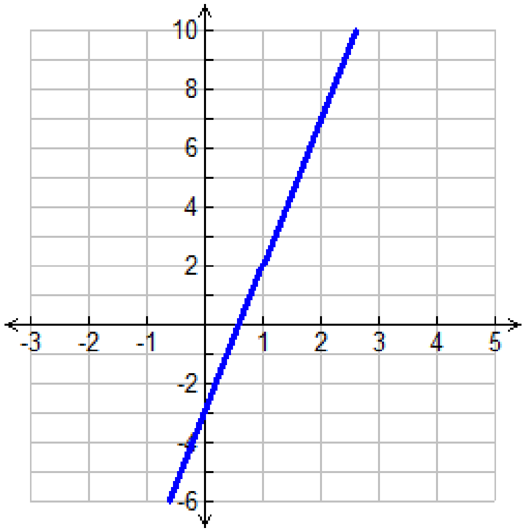

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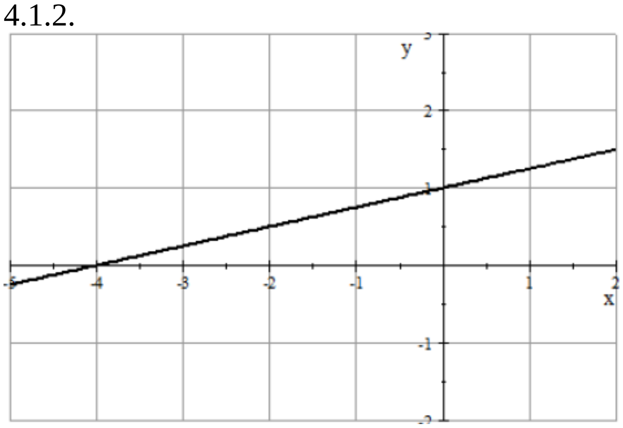

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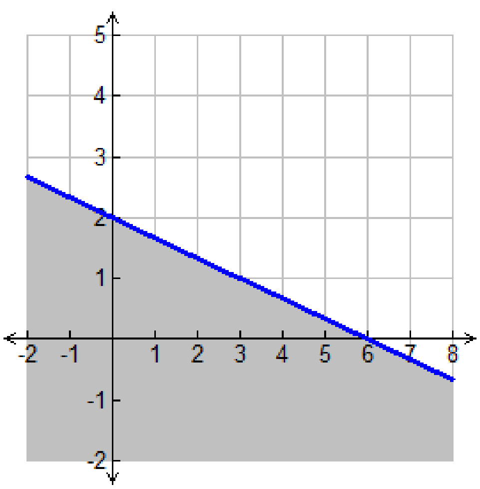

Section 4.1 Exercises

Graph the linear equation by plotting points.

1. $y = 3x - 7$
2. $y = -9x + 12$
3. $3x + y = 10$
4. $y = 5x + 9$

Graph the linear equation using the intercept method.

5. $4x - 2y = 12$
6. $5x - 10y = 20$
7. $x + y = -4$
8. $y = 3x - 15$

Graph the linear inequality.

9. $y > 3x - 9$
10. $y \leq x + 7$
11. $3y - x \leq -6$
12. $10x - 30y \geq 60$

Graph the horizontal or vertical line.

13. $x = -9$
14. $y = 1$

Section 4.1 Exercises – Answer Key

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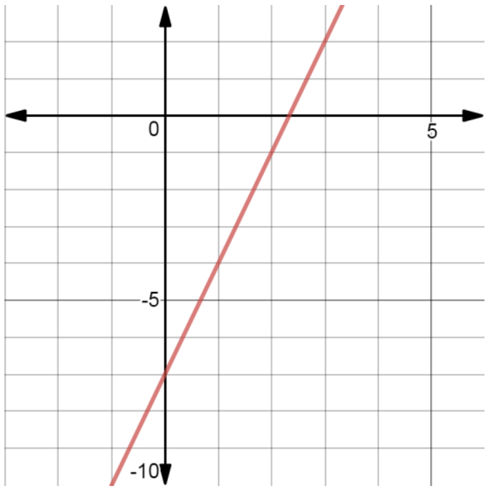

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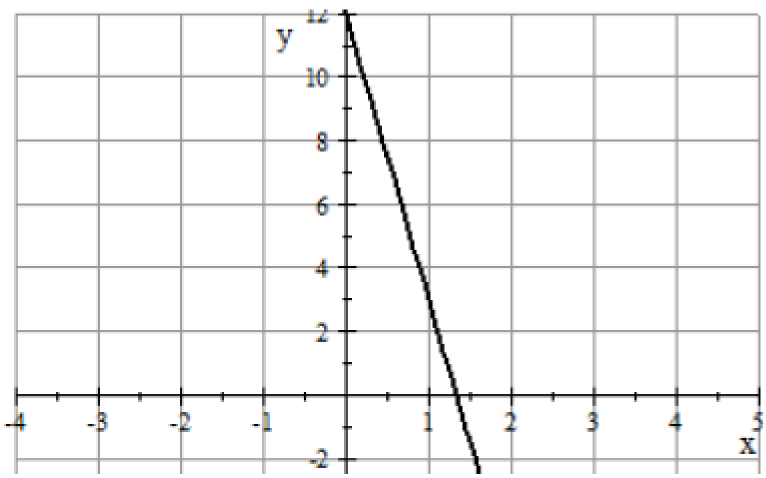

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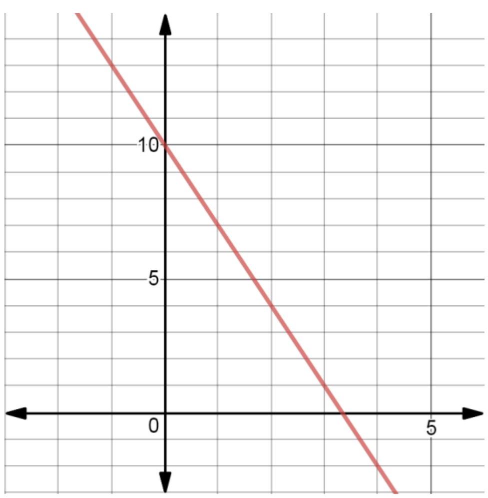

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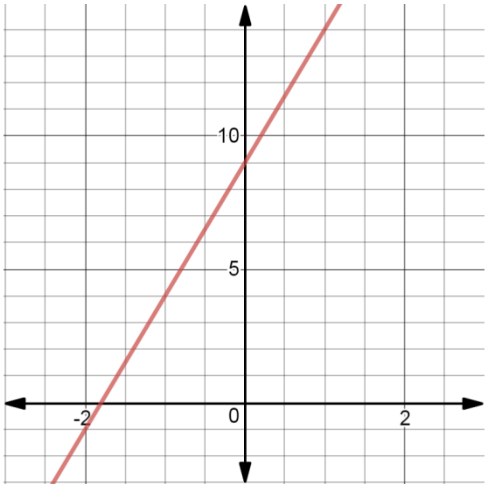

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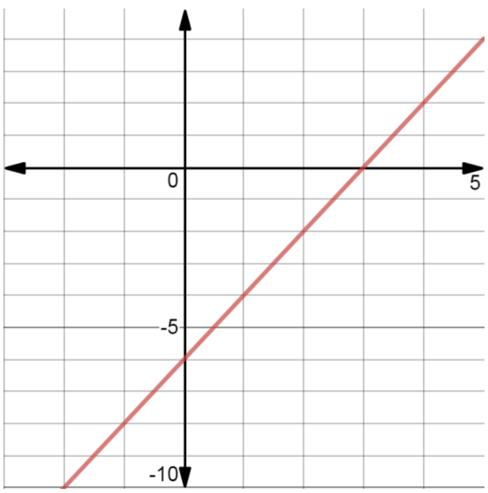

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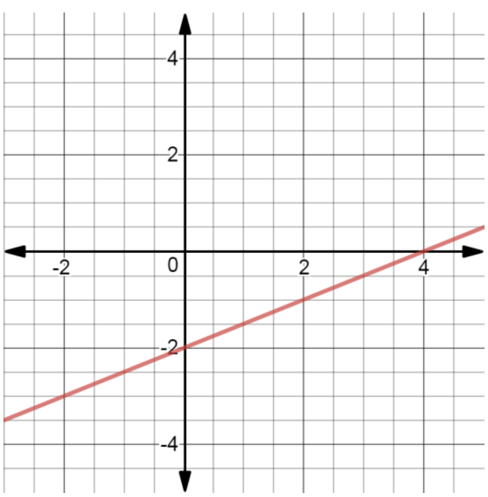

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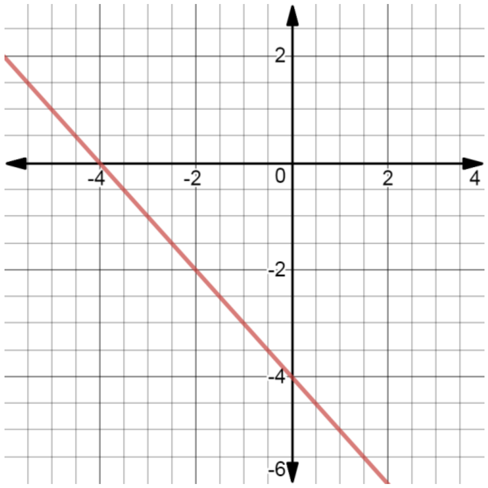

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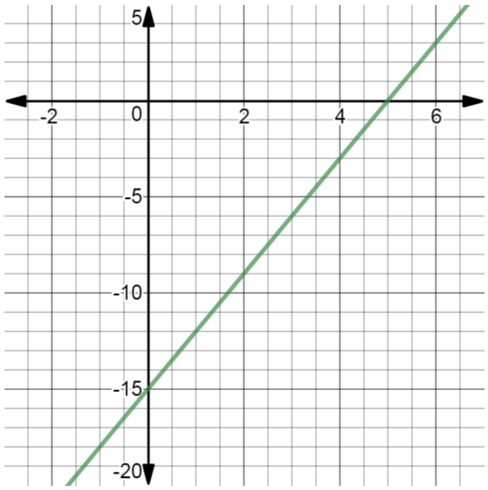

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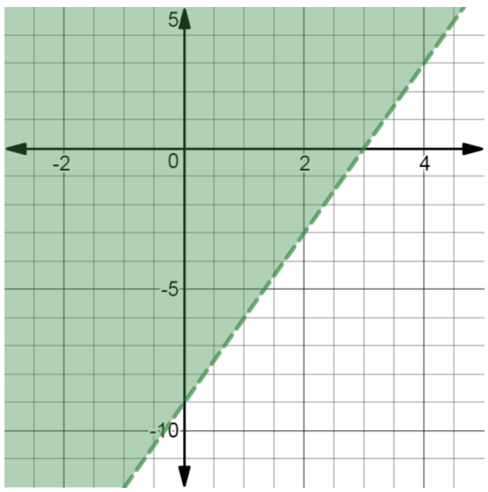

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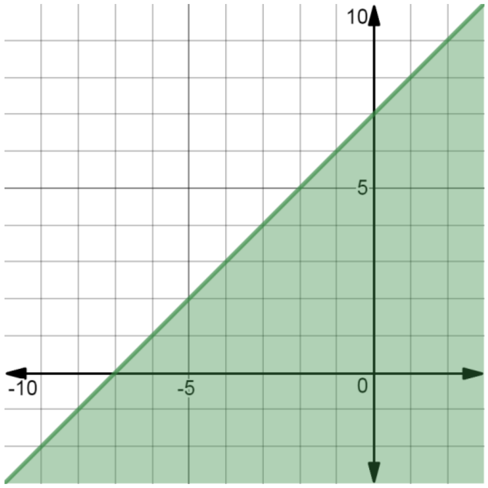

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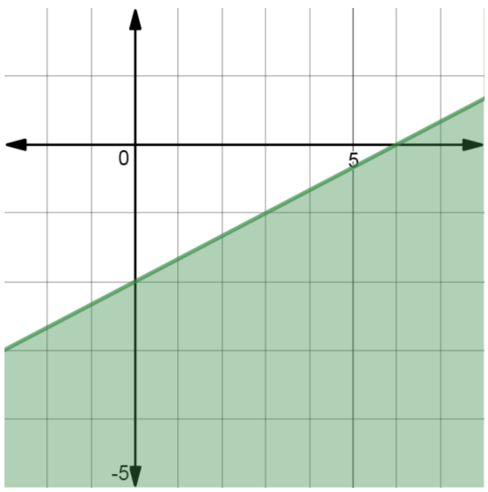

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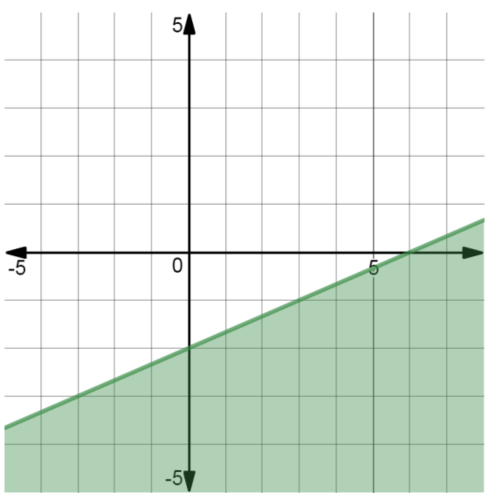

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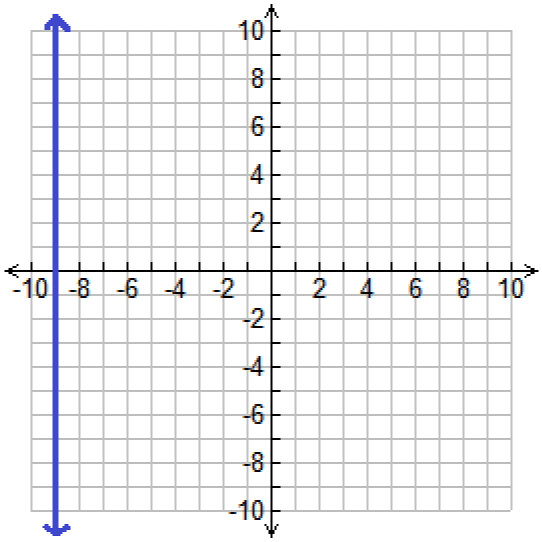

14.