Definition of an Increasing Function and of a Decreasing Function

Def.: A function *f* is *increasing* on an interval if for any two numbers x_1 and x_2 satisfying $x_1 < x_2$,

Def.: A function *f* is *decreasing* on an interval if for any two numbers x_1 and x_2 satisfying $x_1 < x_2$,

Relationships Between a Function and Its First Derivative

Theorem: Let *f* be a function that is continuous on the closed interval $a \le x \le b$ and that is differentiable on the open interval $a < x < b$.

1. If $f'(x) > 0$ for all *x* satisfying $a < x < b$, then *f* is   on $a \le x \le b$.

2. If $f'(x) < 0$ for all *x* satisfying $a < x < b$, then *f* is   on $a \le x \le b$.

3. If $f'(x) = 0$ for all *x* satisfying $a < x < b$, then *f* is   on $a \le x \le b$.

First Derivative Test for Relative Maxima and Minima

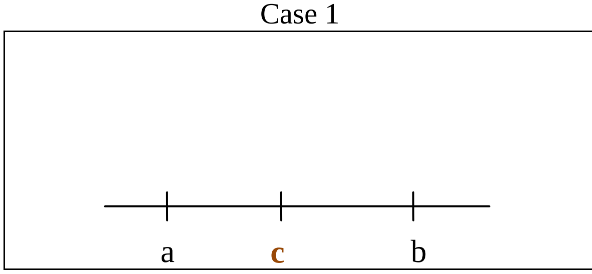
Theorem: Let ***c*** be a critical number of a function *f* that is continuous on an open interval *I* containing ***c***. If *f* is differentiable on *I*, except possibly at ***c***, then the point *(**c** , f(**c**))* can be classified as follows.

1. If $f'(x)$ changes from **negative** to **positive** at ***c***, then *f* has a ***relative***   at *(**c** , f(**c**))*.

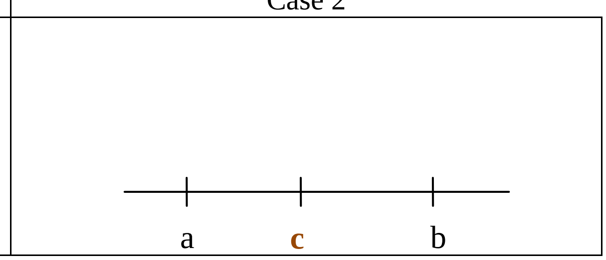
2. If $f'(x)$ changes from **positive** to **negative** at ***c***, then *f* has a ***relative***   at *(**c** , f(**c**))*.

3. If $f'(x)$ is **negative** on both sides of ***c*** or **positive** on both sides of ***c***, then *(**c** , f(**c**))*, is neither a **relative minimum** nor a **relative maximum**.

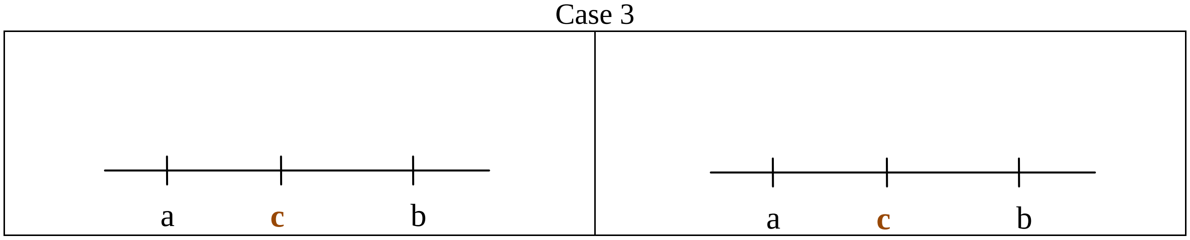
Case 1



Case 2



Case 3



Exercise 1: Determine the critical points of $g(x) = 2x^3 + 3x^2 - 36x + 1$, and use the *First Derivative Test* to **classify** them (as relative **maximums**, **minimums**, or neither.)

First, determine the critical number(s) by solving $g'(x) = 0$ for x.

$g'(x) =$ $\qquad = 6(x^2 + x - 6) =$ $\qquad = 0$

critical numbers: $x =$ critical points: $(-3, g(-3)) =$

$x =$ $(2, g(2)) =$

Next, test the critical numbers using a *sign chart*.

-3 **2**

test *x*:

$g'(x)$:

sign of $g'(x)$:

From the $g'(x)$ information, we **conclude** the following by the *First Derivative Test*:

There is a **relative maximum** at $x =$ because $g'(x)$ changes from

and a **relative minimum** at $x =$ because $g'(x)$ changes from

* * The sign chart also grants us the following information.

g is increasing on and decreasing on

Exercise 2: Determine the critical points of $h(x) = x^3 - 6x^2 + 12x - 3$, and use the *First Derivative Test* to **classify** them (as relative **maximums**, **minimums**, or neither.)

First, determine the critical number(s) by solving $h'(x) = 0$ for x.

$h'(x) =$

critical number: $x =$ critical point:

Next, test the critical number using a *sign chart*.

test x:

$h'(x)$:

sign of $h'(x)$:

From the $h'(x)$ information, we **conclude** the following by the *First Derivative Test*:

* * We can deduce the following from the sign chart and the existence of a single critical number:

Exercise 3 (#60): Restrict the domain of $f(x) = x + 2\sin(x)$ to $0 < x < 2\pi$. Determine the open interval(s) on which *f* is increasing or decreasing, and apply the *First Derivative Test* to identify all relative extrema.

First, determine the critical number(s) by solving $f'(x) = 0$ for *x*.

critical numbers on $0 < x < 2\pi$: $x =$ critical points:

$x =$

Next, test the critical numbers using a *sign chart*.

test *x*:

$f'(x)$:

sign of $f'(x)$:

From the $f'(x)$ information, we **conclude** the following by the *First Derivative Test*.

There is a **relative maximum** at $x =$ because $f'(x)$ changes from

and a **relative minimum** at $x =$ because $f'(x)$ changes from

* * The sign chart also grants us the following information.

f is increasing on and decreasing on