The Arctangent Rule

Since (see page 176 in the text) the derivative of *f(x) = arctan(x)* is then

$$\int \frac{1}{x^2+1}\,dx \;=$$

More generally, $$\int \frac{1}{u^2+a^2}\,du \;=$$

Exercise 1: Determine the following indefinite integral. $\int \frac{1}{x^2+16}\,dx$

Write this integral in the form $\int \frac{1}{u^2+a^2}\,du$ by letting $u =$ and $a =$.

Note then that $du =$

Thus, $\int \frac{1}{x^2+16}\,dx \;=$

Exercise 2: Determine the following indefinite integral. $\int \frac{1}{9x^2+16}\,dx$

Write this integral in the form $\int \frac{1}{u^2+a^2}\,du$ by letting $u =$ and $a =$.

Note then that $du =$

Thus, $\int \frac{1}{9x^2+16}\,dx \;=\; \int \frac{1}{(\quad)^2+(\quad)^2}\,dx \;=\; \frac{1}{3}\int \frac{1}{u^2+a^2}\,du \;=\; \frac{1}{3}\cdot\frac{1}{a}\arctan\left(\frac{u}{a}\right)+c$

$=$ $=$

Exercise 3: Determine the following indefinite integral. $\int \frac{x}{9x^4+50}\,dx$

$$\int \frac{x}{9x^4+50}\,dx = \int \frac{x}{(\quad)^2+(\quad)^2}\,dx$$

Write this integral in the form $\int \frac{1}{u^2+a^2}\,du$ by letting $u =$ and $a =$.

Note then that $du =$

Thus, $\int \frac{x}{9x^4+50}\,dx = \int \frac{x}{(\quad)^2+(\quad)^2}\,dx =$

Exercise 4: Which of the following two integrals requires the arctangent rule, and which requires nothing more than basic *u*-substitution? Determine each indefinite integral.

$\int \frac{x^3}{64x^8+4}\,dx$

$u =$ $\Rightarrow$ **$du =$** Also,

$\int \frac{x^3}{64x^8+4}\,dx =$

$\int \frac{x^7}{64x^8+4}\,dx$

$u =$ $\Rightarrow$ **$du =$**

$\int \frac{x^7}{64x^8+4}\,dx =$

The Arcsine Rule

Since (see page 176 in the text) the derivative of $f(x) = arcsin(x)$ is then

$$\int \frac{1}{\sqrt{1-x^2}}\,dx \ =$$

More generally, $$\int \frac{1}{\sqrt{a^2-u^2}}\,du \ =$$

Exercise 5: Determine the following indefinite integral. $$\int \frac{x^5}{\sqrt{49-x^{12}}}\,dx$$

Write this integral in the form $\int \frac{1}{\sqrt{a^2-u^2}}\,du$ by letting $u =$ and $a =$.

Note then that $du =$

Thus, $$\int \frac{x^5}{\sqrt{49-x^{12}}}\,dx \ = \ \int \frac{x^5}{\sqrt{49-x^{12}}}\,dx$$

$$= \frac{1}{6}\int \frac{1}{\sqrt{a^2-u^2}}\,du \ = \ \frac{1}{6}\arcsin\left(\frac{u}{a}\right)+c \ =$$

Exercise 6: Determine the following definite integral. $\int_0^1 \frac{x+5}{\sqrt{9-x^2}}\,dx$

The numerator of the integrand looks like it will be **trouble**. However, since the numerator is a sum, we can rewrite the integrand as the sum of two fractions.

Thus, $\int_0^1 \frac{x+5}{\sqrt{9-x^2}}\,dx \quad = \quad \int_0^1 \frac{x}{\sqrt{9-x^2}}\,dx \quad + \quad \int_0^1 \frac{5}{\sqrt{9-x^2}}\,dx$

$u =$ $\Rightarrow$ $du =$ $u =$ $\Rightarrow$ $du =$ Also, $a =$

$x = 1 \Rightarrow u =$ $x = 1 \Rightarrow u =$

$x = 0 \Rightarrow u =$ $x = 0 \Rightarrow u =$

So, $\int_0^1 \frac{x+5}{\sqrt{9-x^2}}\,dx \quad = \quad \int_0^1 \frac{x}{\sqrt{9-x^2}}\,dx \quad + \quad 5\int_0^1 \frac{1}{\sqrt{9-x^2}}\,dx$

The Arcsecant Rule

$$\int \frac{1}{u\sqrt{u^2 - a^2}}\, du \ =$$

Exercise 7 (#12): Determine the following definite integral. $\int \frac{2}{x\sqrt{9x^2 - 25}}\, dx$

SURGEON GENERAL'S WARNING: CALCULUS Causes Lung Cancer, Heart Disease, Emphysema, and May Complicate Pregnancy.

Surgeon General's Warning: You may have to use your bag of algebraic and trigonometric tricks—or go to **WalMarth** and buy new ones—in order to be able to rewrite an integrand in a form that is recognizable for integration. We will look at a few of the tricks via some even exercises from the text.

Exercise 8: $\int \frac{10}{4t^2 - 4t + 5}\, dt$

Try *u*-substitution: $u =$ $\Rightarrow$ $du =$

Trouble: The numerator

Instead, start off by **completing the square** in the denominator:

$$4t^2 - 4t + 5 =$$

Thus, $\int \frac{10}{4t^2 - 4t + 5}\, dt = \int \frac{10}{(2t-1)^2 + 2^2}\, dt$ Let ***u* =** Then ***du* =**

Also, let ***a* =**

Then, $\int \frac{10}{(2t-1)^2 + 2^2}\, dt = \int \frac{}{(2t-1)^2 + 2^2}\, dt = \int \frac{2}{(2t-1)^2 + 2^2}\, dt$