HW3-6 Soln)

Let the origin be at the player's forward foot (right under the ball at release) and let up be +y and to the right be +x.

$x_i = 0$ m $\qquad y_i = 2.5$ m

$x_f = 12$ m $\qquad y_f = 6$ m

$v_{xi} = v_o \cos\theta_o = 16\cos\theta_o \qquad v_{yi} = v_o \sin\theta_o = 16\sin\theta_o$

$v_{xf} = 16\cos\theta_o \qquad v_{yf} = ?$

$a_x = 0$ m/s^2 $\qquad a_y = -10$ m/s^2

$t = ?$

(3)

$$x = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \rightarrow x = 0 + v_o\cos\theta_o\, t + 0$$

$$t = \frac{x}{v_o\cos\theta_o}$$

(3)

$$y = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = y_i + v_o\sin\theta_o\, t + \frac{1}{2}a_y t^2 = y_i + v_o\sin\theta_o\frac{x}{v_o\cos\theta_o} + \frac{1}{2}a_y\left(\frac{x}{v_o\cos\theta_o}\right)^2$$

$$y = y_i + x\tan\theta_o + \frac{a_y x^2}{2v_o^2}\left(\frac{1}{\cos\theta_o}\right)^2 = y_i + x\tan\theta_o + \frac{a_y x^2}{2v_o^2}(\tan^2\theta_o + 1)$$

$$= \left(y_i + \frac{a_y x^2}{2v_o^2}\right) + x\tan\theta_o + \left(\frac{a_y x^2}{2v_o^2}\right)\tan^2\theta_o$$

This is quadratic in the tangent. Re-arrange.

$$\left(\frac{a_y x^2}{2v_o^2}\right)\tan^2\theta_o + (x)\tan\theta_o + \left(y_i - y + \frac{a_y x^2}{2v_o^2}\right) = 0$$

$$\left(\frac{(-10)12^2}{2(16^2)}\right)\tan^2\theta_o + (12)\tan\theta_o + \left(2.5 - 6 + \frac{(-10)12^2}{2(16^2)}\right) = 0$$

$$(-2.8125)\tan^2\theta_o + (12)\tan\theta_o + (-6.0125) = 0$$

$$\tan\theta_o = \frac{-12 \pm \sqrt{12^2 - 4(-2.8125)(-6.0125)}}{2(-2.8125)} = \begin{matrix} 3.687 \\ 0.580 \end{matrix}$$

$$\theta_{o1} = \arctan(0.580) = 30.1^o$$

$$\theta_{o2} = \arctan(3.687) = 74.8^o$$