

HW 11-6 Soln)

First, find the effective spring constant, k_{eff} . That is, we replace the two springs with a single spring that exerts the same force for the same amount of stretching as do the two springs.

Both springs exert forces on the mass: $F_1 = -k_1\Delta x$ and $F_2 = -k_2\Delta x$

The total force $F = F_1 + F_2 = -k_1\Delta x + -k_2\Delta x = -(k_1 + k_2)\Delta x = -k_{\text{eff}}\Delta x$

Comparison of these results gives that $k_{\text{eff}} = k_1 + k_2$

Now, we know that

$$f_o = \frac{1}{2\pi} [k_{\text{eff}}/m]^{1/2} = \frac{1}{2\pi} [(k_1 + k_2)/m]^{1/2} = \frac{1}{2\pi} [(300 + 900)/7]^{1/2} = 2.08 \text{ Hz}$$

For the really interested:

One might say that this makes sense if the two springs have the same equilibrium point, but what if they don't? Actually it doesn't matter:

Let y_o be the eq. pt for spring 1 and z_o be the eq. pt for spring 2. Let's assume that $y_o > z_o$, although it doesn't really matter, since the reverse case will introduce two negative signs which will cancel one another anyway. If we hook them together as in the diagram, the system will come to a new equilibrium point we'll call x_o , at which one spring is compressed and the other stretched so that the magnitudes of the two spring forces will be equal:

$$k_1(y_o - x_o) = k_2(x_o - z_o)$$

We can re-arrange this into something which will eventually be useful:

$$(k_1 + k_2)x_o = (k_1y_o + k_2z_o)$$

Now, let's displace the mass to some new position x . The total force on the box will be:

$$F_{\text{tot}} = k_1(x - y_o) + k_2(x - z_o)$$

$$F_{\text{tot}} = k_1x - k_1y_o + k_2x - k_2z_o$$

$$F_{\text{tot}} = (k_1 + k_2)x - (k_1y_o + k_2z_o)$$

From above, we remember that the last term is $(k_1 + k_2)x_o$

$$F_{\text{tot}} = (k_1 + k_2)x - (k_1 + k_2)x_o = (k_1 + k_2)(x - x_o), \text{ as we suspected.}$$