HW 9-10 Soln)

Forces on the wheel: weight, force for the axle, and the tension.
Forces on the hanging mass: weight and tension.

W_g – conservative
$W_{\text{axle force}}$ – no displacement of the wheel, so no work done
W_T – although the tension does positive work on the wheel and negative work on the hanging mass, these terms exactly cancel to zero.

Let the hanging mass start at y = 0.

So, $W_{NC} = 0$. Then,

$$0 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + \frac{1}{2}Mv_f^2 - \frac{1}{2}Mv_i^2 + \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 + gmy_f - gmy_i + gMy_f - gMy_i$$

The terms in red are either zero of exhibit no change over the duration of the problem.

$$0 = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + gmy_f$$

Similarly to the rolling without slipping problems, the wheel lets out string such that its angular speed is correlated to the translational sped of the hanging mass:

$$v = R\omega\ .$$

Then,

$$0 = \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_f}{R}\right)^2 + gmy_f$$

$$\left(\frac{1}{2}m + \frac{1}{4}M\right)v_f^2 = -gmy_f$$

$$v_f = \sqrt{\frac{-gmy_f}{\frac{1}{2}m + \frac{1}{4}M}} = \sqrt{\frac{-10(0.5)(-3)}{\frac{1}{2}0.5 + \frac{1}{4}1.8}} = 4.63\ \text{m/s}$$