Section 1-10 - Static Equilibrium

DISCUSSION 10-1

Suppose you're constructing some structure, a bridge or office building. It's composed of many parts, none of which should move. What condition should be imposed on an object, such as a steel beam or a brick, so that it doesn't move? What else shouldn't the object do? What condition must be met for that motion to be avoided?

Conditions for Static Equilibrium

Statics is a sub-topic of physics and engineering which is incredibly important; mechanical engineers study it for many semesters because of its applications to the design of buildings and other structures. What is needed are the conditions under which an object or assemblage of objects will not shift. These boil down to:

$$\sum_n \vec{F}_n = 0 \quad \text{and} \quad \sum_n \vec{\tau}_n = 0 \quad .$$

That is, we want the objects not to accelerate or rotate. And, that is it for Section 10.

I'm sure, however, that you'd like a few examples. Our convention for this chapter is that we shall write the magnitudes of the force and r vectors, then add the appropriate signs in front of each torque term; positive for torques out of the page (that is, those which would act to accelerate the object CCW), and negative for torques into the page (those which would act to accelerate the object CW).

In the previous section, we considered rotations about specific axes. In this section, the object is not rotating about <u>any</u> axis, that is, the torque about any axis you may care to choose will be zero. This gives you a fair amount of freedom in solving problems. In this section, the point about which the torques are calculated is called the *pivot point*. My suggestion is to write the toque equation first, choosing if possible a pivot that makes the torques due to forces of unknown magnitude zero. In that way, many problems can be solved by considering only the torque equation without involving the force equations.

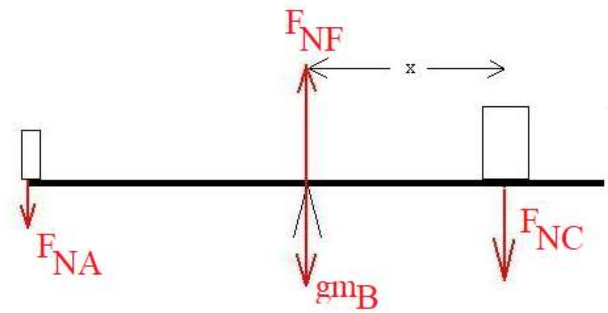
Remember from Derivation 9-1 that we can act as if all of the weight of an object is applied at the object's center of mass.

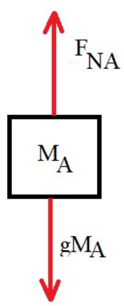
EXAMPLE 10-1

Consider a see-saw of length L (6m) and mass m (m = 15 kg) which is pivoted at the center. Anna (m_A = 20kg) sits right at the end of the board. Where should Carli (m_C = 35 kg) sit so that the board is balanced?

Much as we did back in Section 5, we need to choose an object (or objects) to analyze; in this case, we choose the board. Let's draw a free body diagram (with +y upward and +x to the right) to inventory the forces:

F_{NF} is the normal force exerted by the fulcrum of the see-saw on the board, and gm_B is the weight of the board, which we assume can be thought to act as the center of the board. The forces labelled F_{NA} and F_{NC} are <u>not</u> the weights of Anna and Carli, but rather normal forces exerted by them on the board; remember that the weight of each child <u>acts on the child</u>. Through the use of NII and NIII, these forces are numerically equal to those weights.





For Anna, we have from NII that

$$+F_{NA} - gM_C = 0$$

$$+F_{NA} = gM_C = 10(20) = 200\ N\ .$$

The exact same calculation for Carli results in F_{NC} = 350 N.

Let's do the problem twice to illustrate a point.

1) Choose the left end of the board as the pivot. The torque requirement is[1]

$$\sum_n \vec{\tau}_n = (0)F_{NA}\sin(?) + (3)(F_{NF})\sin(90^o) - (3)(gm_B)\sin(90^o)$$
$$- r_{LEFT\ END}(F_{NC})\sin(90^o) = 0.$$

We're going to need to know F_{NF}, so we'll go to the vertical force equation:

$$\sum_n \vec{F}_{nx} = 0 \quad \text{and} \quad \sum_n \vec{F}_{ny} = +F_{NF} - F_{NA} - F_{NC} - gm_B = 0\ ,$$

$$F_{NF} = F_{NA} + F_{NC} + gm_B = 200 + 350 + 150 = 700\ N\ .$$

Returning to the torque equation,

$$(3)(F_{NF}) - (3)(gm_B) = r_{LEFT\ END}(F_{NC}).$$

[1] Note the question mark inserted as the angle. Since r = 0 for that force about that pivot, the angle is undefined. However, I think that keeping the format of each term uniform lessens the probability of making an error and at the same time, indicates to me that you know that that term is zero and why.

$$r_{\text{LEFT END}} = \frac{3F_{NF} - 3gm_B}{F_{NC}} = \frac{3(700) - 3(150)}{350} = \text{4.71 m from the left end}.$$

2) Choose the center of the board as the pivot. The torque equation is then

$$\sum_n \vec{\tau}_n = +(3)F_{NA}\sin(90^o) + (0)(F_{NF})\sin(?) - (0)(gm_B)\sin(?)$$

$$- r_{\text{CENTER}}(F_{NC})\sin(90^o) = 0 \; ;$$

$$(3)F_{NA} = r_{\text{CENTER}}(F_{NC}) \; ;$$

$$r_{\text{CENTER}} = \frac{(3)F_{NA}}{F_{NC}} = \frac{3(200)}{350} = \text{1.71 m to the right of the center},$$

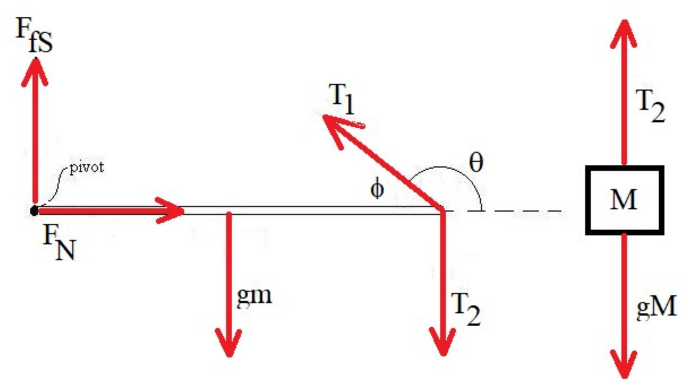
which is of course the same answer.

What we see, then, is that a judicious choice of the pivot such as in the second solution can save a great deal of effort.

EXAMPLE 10-2

Consider a horizontal uniform beam of mass m and length L supporting a sign of mass M. The beam is attached to the wall with a wire which makes an angle θ with the beam. Its other end is supported by the friction between the end of the beam and the wall. How large would the co-efficient of static friction need to be to keep the beam from slipping?

Again using the usual coördinate system of +y up and +x to the right, and picking the left end of the beam as the pivot, we write that:



$$+T_2 - gM = 0 \; ;$$

$$+F_N - T_1\cos\phi = 0 \; ;$$

$$F_{fS} - gm + T_1\sin\phi - T_2 = 0 \; ;$$

$$F_{fS} = \mu_S F_N \; (\text{crit. sit.}) \; ;$$

and

$$(0)F_{fS}\sin(?) + (0)F_N\sin(?) - \frac{L}{2}gm\sin 90^o - LT_2\sin 90^o + LT_1\sin\phi = 0 \; .$$

Simplifying the torque equation and substituting for T_2 results in

$$T_1 \sin\phi = \frac{gm}{2} + T_2 = \frac{gm}{2} + gM \;.$$

Now it's just substitution:

$$\mu_S = \frac{F_{fS}}{F_N} = \frac{gm - T_1 \sin\phi + T_2}{T_1 \cos\phi} = \frac{gm - \left(\frac{gm}{2} + gM\right) + gM}{\left(\frac{\frac{gm}{2} + gM}{\sin\phi}\right)\cos\phi} = \frac{m \tan\phi}{m + 2M}.$$

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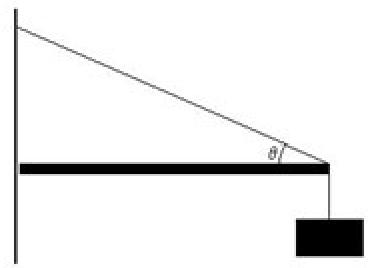
HOMEWORK 10-1

A sign weighing 300 N is suspended at the end of a massless beam 2 m in length, as shown. The beam is attached to the wall with a hinge, and its other end is supported by a wire. What is the tension in the wire if it makes an angle of 30° with the beam?

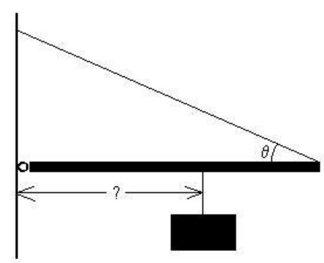


HOMEWORK 10-2

A sign weighing 500 N is suspended at the end of a uniform 300 N beam 5 m in length, as shown. The beam is attached to the wall with a hinge, and its other end is supported by a wire. What is the tension in the wire if it makes an angle of 25° with the beam? Use the same figure as for Homework 10-1.

HOMEWORK 10-3

A sign weighing 200 N is suspended under a uniform 300 N beam 4 m in length, as shown. The beam is attached to the wall with a wire which makes a 53° angle with the beam. Its other end is supported by a hinge connected to the wall. If the maximum tension permissible in the wire is 300 N, what is the range of distances from the wall that the sign can hang without causing the wire to snap?



HOMEWORK 10-4

An 6m long, 110N ladder rests against a smooth wall. The co-efficient of static friction between the ladder and the ground is 0.8, and the ladder makes a 53° angle with the ground. How far up the ladder can an 800N person climb before the ladder starts to slip?