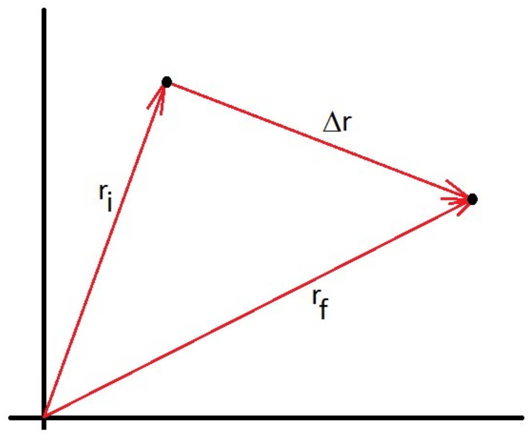
Section 1-2 - Kinematics in One Dimension

Kinematics is the study of the motion (same root as *cinema*) of an object, without regard to the causes of that motion. Most of this section involves defining a number of terms.

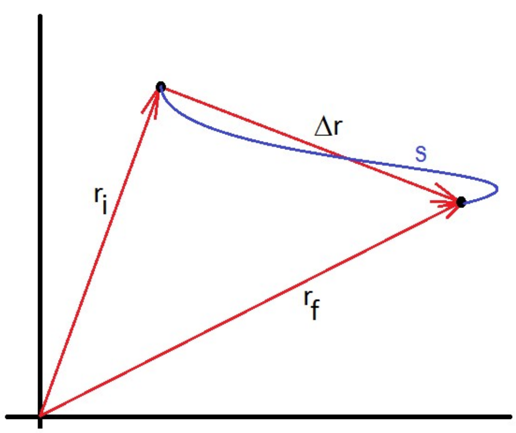
Displacement and Distance



We'll need first of all to be able to define the *location* or *position* of an object. We talked in the last section about how to describe the location of an object in terms of its x-, y-, and z-coördinates. Alternately, we can describe the position using spherical coördinates, using r, θ, and φ, or really with any of a large number of systems. Let the vector $\vec{r}$ point from the origin to the location of the object. If the object moves, then there will be an initial position vector, $\vec{r}_i$, and a final position vector, $\vec{r}_f$. The difference between them will be $\Delta\vec{r}$. We say it's the difference because, from the figure and using the tail-to-tip method, $\vec{r}_f = \vec{r}_i + \Delta\vec{r}$, and so

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i \quad .$$

The vector $\Delta\vec{r}$ is called the *displacement*; it depends only on the initial position and the final position, and not on the path the object took between them.



Suppose that the blue line represents the actual path taken by the object. The *distance* s is the length of the actual path taken. As an analogy, think of it as being like the number of steps taken. Distance is a scalar quantity. Consider three scenarios: 1) the object moves in a straight line from its initial to its final position; 2) the object moves along the blue line shown; 3) the object takes a trip to the moon and returns to earth to end at its final location. While the distances in each case are different, the displacement in each case is the same as for the other situations. Note this special case, however: if the object travels along a straight line without reversing direction, the distance and the magnitude of the displacement are the same value.

DISCUSSION 2-1

Suppose that an object starts out at $x_i = 3$ m and ends at $x_f = 5$m, and makes that trip smoothly and without reversing direction. What is the displacement? Now, suppose instead that the object travels from x = 3 m to x = 15 m, then back to x = - 8 m, then on to x = 5 m. What is the displacement in that case?

Suppose instead that the object moved from x = 5 m to x = 3 m. What then would be the displacement? Is the displacement a scalar or vector quantity? [1]

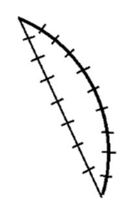
Suppose that Object 1 moves from x = 5 m to x = 9 m, while Object 2 moves from x = 7 m to x = 11 m. Which object had the larger displacement?

In the first two cases, the displacements are the same at +2 m. As was mentioned, the path is not relevant to the displacement. In the third case, the displacement is -2 m. Since there is a difference between the motion 2 meters to the right and motion 2 meters to the left, displacement must be a vector. Finally, both Objects 1 and 2 have the same displacements. In the next few sections, we will let the sign of a vector indicate its direction.

Distance (s) is the term we use for the length of the path taken. As a rough analogy, think of the distance as the number of steps one takes getting from A to B.

DISCUSSION 2-2

What relationship exists between the distance and the magnitude of the displacement? Suppose that I walk the 7 meters from the desk to the back of the room. What is the magnitude of my displacement? What is my distance? Now, however, I walk from the front of the room to the rear, then back to the front, then return to the rear. What is the magnitude of my overall displacement? What is my overall distance? Can you explain why these results are different?



No matter what I do, any motion adds to my distance travelled (I'm taking steps). On the other hand, if walking toward the rear of the room is positive displacement, walking toward the front is negative displacement that cancels the first part of my trip, and eventually I return to the front from where I started for a total displacement of zero. Similarly, if I travel along a circular arc path, the distance and the magnitude of my displacement will be different. So long as the direction of motion doesn't change, the distance is the same as the magnitude of the displacement.

EXERCISE 2-1

Suppose that an object starts out at $x_i = 3$ m and ends at $x_f = 5$ m, and makes that trip smoothly and without reversing direction. What is the distance? Now, suppose instead that the object travels from x = 3 m to x = 15 m, then back to x = - 8 m, then on to x = 5 m. What is the distance in that case?

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[1] So, if displacement is a vector, it follows that position must technically also be a vector. But how can an object be described as being at x = 5 m in any particular direction? I think we must consider position as being relative to the origin, somewhat awkwardly, as a virtual displacement.

Velocity and Speed

Often, we want to know how quickly an object gets from one spot to another. If we say that the object is at position $\mathbf{r}_i$ at time t_i, and arrives at position $\mathbf{r}_f$ at time t_f, then we can define the *average velocity* to be the displacement *per* unit time, or

$$\vec{v}_{AVE} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} = \frac{\Delta\vec{r}}{\Delta t} .$$

DISCUSSION 2-3

Is the average velocity a vector or a scalar?

EXAMPLE 2-1

Find the average velocity for an object moving from x = 3m to x = 5m in 3 seconds.

$$v_{AVE} = \frac{x_f - x_i}{t_f - t_i} = \frac{5-3}{3-0} = +0.67 \text{ m/s} .$$

Find the average velocity for an object moving from x = 5m to x = 3m in 3 seconds.

$$v_{AVE} = \frac{x_f - x_i}{t_f - t_i} = \frac{3-5}{3-0} = -0.67 \text{ m/s} .$$

Note that the sign indicates that the velocity points to the left (or more correctly, in the negative x direction).

Find the average velocity for an object moving from x = 3 to x = 15, then back to x = - 8, then on to x = 5 in 3 seconds.

$$v_{AVE} = \frac{x_f - x_i}{t_f - t_i} = \frac{5-3}{3-0} = +0.67 \text{ m/s} .$$

This result may seem a bit strange in that a person doing the first motion could do so in a leisurely manner, while someone performing the third would be zipping back and forth in a superhuman way. Words that may seem to mean the same thing in everyday speech can mean very different things in physics, according to how we define them. For example, …

We define the *average speed* as the distance covered divided by the time interval over which the motion happened. There is no special symbol for average speed.

$$\text{average speed} = \frac{s}{t_f - t_i}$$

EXAMPLE 2-2

Find the average speed for an object moving from x = 3m to x = 5m in 3 seconds.

$$v_{AVE} = \frac{s}{t_f - t_i} = \frac{2}{3-0} = 0.67 \text{ m/s} .$$

Find the average speed for an object moving from x = 5m to x = 3m in 3 seconds.

$$v_{AVE} = \frac{s}{t_f - t_i} = \frac{2}{3-0} = 0.67 \text{ m/s} .$$

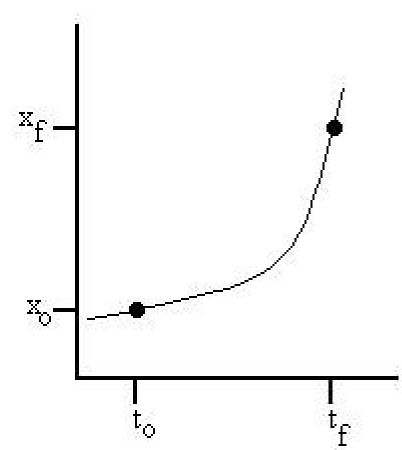
Find the average speed for an object moving from x = 3 to x = 15, then back to x = - 8, then on to x = 5 in 3 seconds.

$$v_{AVE} = \frac{s}{t_f - t_i} = \frac{48}{3-0} = 16 \text{ m/s} .$$

The average velocity discussed above is considered over an interval of time. How can we find the *instantaneous velocity*, the velocity at an instant of time? Well, let's consider the average velocity over some interval, but then make the interval shorter and shorter, until it is as close to a single moment as possible. Mathematically, that is,

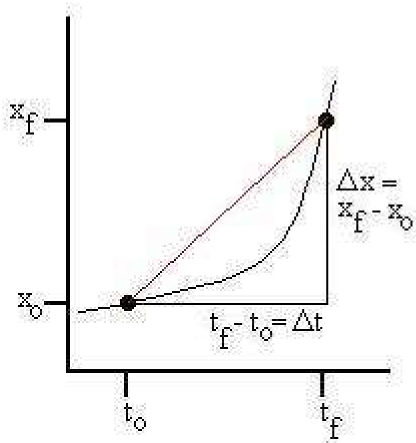
$$v_{INST} = \lim_{t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} .$$

Alternately, consider the following graph, which shows the position of an object as a function of time, x(t):



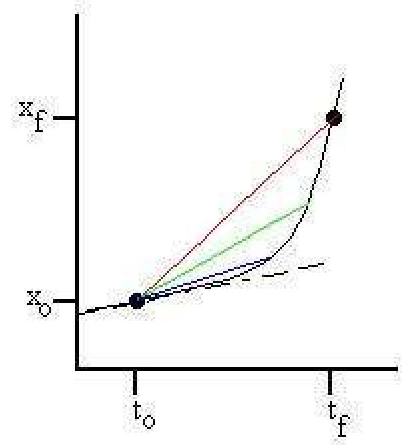
How would the average velocity between t_i and t_f be represented on this graph? Consider the second graph.

$$v_{AVE} = \frac{x_f - x_i}{t_f - t_i} = \frac{\text{rise}}{\text{run}} .$$



This quantity is the slope of the red line connecting the two points. How to find the velocity at time t_i? Let's decrease the interval, Δt (see the third graph). As the interval becomes smaller, the average velocity approaches the instantaneous velocity, or graphically, the slope representing the average velocity approaches the slope of the line tangent to the x(t) curve at the point at which we wish to know v(t). Mathematically, we write this as

$$v_{INST} = \lim_{t\to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \ .$$



We can also talk about the *instantaneous speed*:

$$\text{Inst. speed} = \lim_{t\to 0} \frac{s}{\Delta t} \ .$$

Now, we see that when the time interval becomes sufficiently short, the object has no opportunity to deviate from a straight segment on its path. In that case, as discussed above, the distance and the magnitude of the displacement become equal. So,

$$\text{Inst. speed} = \lim_{t\to 0} \frac{s}{\Delta t} = \lim_{t\to 0} \frac{|\Delta \vec{r}|}{\Delta t} = |v_{INST}| \ .$$

So the instantaneous speed is the same as the magnitude of the instantaneous velocity.

EXAMPLE 2-3

The position of an object is given by $x(t) = 3t^3 - 2t + 5 - 3\cos(\pi t)$, where the numerical factors are adjusted so that time is in seconds and the position is in meters. Find the velocity at t equals 2 seconds.

$$v(t) = \frac{dx}{dt} = 9t^2 - 2 + 0 - 3(-\pi \sin(\pi t))$$

$$v(2) = 9(4) - 2 + 3\pi \sin(2\pi) = 34\frac{m}{s} .$$

Acceleration (and so on)

Occasionally, we would like to know how quickly the velocity of an object is changing. We define the *average acceleration* as the change in velocity per unit time,

$$\vec{a}_{AVE} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\Delta \vec{v}}{\Delta t} \ ,$$

and the *instantaneous acceleration* as

$$\vec{a}_{INST} = \lim_{t\to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2} .$$

Graphically, the instantaneous acceleration will be the slope of the line tangent to a velocity v. time curve.

The analysis is the same for **a** as it was for **v**, so the work will not be repeated here.

We can continue the process indefinitely. For example, the *jerk* is defined as

$$\vec{\mathrm{J}} = \frac{d\vec{\mathrm{a}}}{dt} = \frac{d^3\vec{\mathrm{r}}}{d\mathrm{t}^3} \;;$$

the *kick* as

$$\vec{\mathrm{K}} = \frac{d\vec{\mathrm{J}}}{dt} = \frac{d^4\vec{\mathrm{r}}}{d\mathrm{t}^4} \;;$$

and the *lurch* as

$$\vec{\mathrm{L}} = \frac{d\vec{\mathrm{K}}}{dt} = \frac{d^5\vec{\mathrm{r}}}{d\mathrm{t}^5} \;.$$

And, of course, there's no reason to stop there.

The acceleration, jerk, kick, and lurch are all vector quantities.

HOMEWORK 2-1

> The position of an object is given by $x(t) = 3t^3 - 2t + 5 - 3\cos(\pi t)$, where the numerical factors are adjusted so that time is in seconds and the position is in meters. Find the acceleration at t equals 2 seconds.

Kinematic Equations for the Special Case of Constant Acceleration

Let's use these definitions to derive some possibly useful relationships. To make life a bit easier, we shall assert the following:

- that the acceleration is constant, at least over some interval in which we are interested (*i.e.*, this is a <u>special case</u>).
- that the problem starts at $t_i = 0$, so that we can just drop that term. We can always put it back, if necessary.
- that all final quantities (t_f, $\vec{r}_f$, $\vec{v}_f$) can be replaced with the more general corresponding variables (t, $\vec{r}$, $\vec{v}$).

In this way, for example, $\Delta t = t_f - t_i = t$.

Start with the definition of the acceleration. Since the acceleration is constant, that value is also the average acceleration. As an analogy, suppose everyone in the class earned a 78 on an exam. What would the average grade be?

Simplify:

$$\vec{a}_{AVE} = \vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\vec{v} - \vec{v}_i}{t}.$$

Re-arrange:

$$\vec{v} = \vec{v}_i + \vec{a}t \ . \ (1)$$

Next, we'll start with the definition of the velocity,

$$\vec{v} = \frac{d\vec{r}}{dt} \quad \rightarrow \quad d\vec{r} = \vec{v}\, dt \ .$$

Eq. (1) gives us the velocity as a function of time, so we can substitute:

$$d\vec{r} = \vec{v}\, dt = (\vec{v}_i + \vec{a}t)\, dt\, .$$

Next. we'll integrate, making sure that the limits of integration for each variable match, *i.e.* we start at $\mathbf{r}_i$ at t = 0, and end at $\mathbf{r}_f$ at time t.

$$\int_{\vec{r}_i}^{\vec{r}_f} d\vec{r} = \int_0^t (\vec{v}_i + \vec{a}t)\, dt \ ,$$

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i = \vec{v}_i t + \tfrac{1}{2}\vec{a}t^2 \ ,$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \tfrac{1}{2}\vec{a}\, t^2\, . \ (3)$$

Next, we'll combine two definitions by taking the dot products of each side of one with the respective sides of the other:

$$\vec{v} = \frac{d\vec{r}}{dt} \ \text{ and } \vec{a} = \frac{d\vec{v}}{dt}.$$

So,

$$\vec{v} \cdot \frac{d\vec{v}}{dt} = \vec{a} \cdot \frac{d\vec{r}}{dt} \ \rightarrow \ \vec{v} \cdot d\vec{v} = \vec{a} \cdot d\vec{r} \ \rightarrow \ v\, dv = \vec{a} \cdot d\vec{r} \ .$$

This last step simplifying the dot product of the velocity terms may seem a bit iffy, but I will show you the process from one step to the next that justify it in Note One at the end of the section.

Let's integrate, again being sure that the limits on each side correspond. Remember we're requiring that $\vec{a}$ is constant, so it can be pulled out of the integral:

$$\int_{\vec{v}_i}^{\vec{v}_f} v\, dv = \vec{a} \cdot \int_{\vec{r}_i}^{\vec{r}_f} d\vec{r} \quad ,$$

$$\frac{1}{2} v_f^2 - \tfrac{1}{2} v_i^2 = \vec{a} \cdot (\vec{r}_f - \vec{r}_i) \quad ,$$

$$v^2 = v_i^2 + 2\,\vec{a} \cdot \Delta\vec{r} \quad . \quad (4)$$

Now, let's swing back for Eq (2). Remember that we have let $t_i = 0$, so $\Delta t = t$.

$$\vec{v}_{AVE} = \frac{\Delta\vec{r}}{t} = \frac{\vec{v}_i t + \frac{1}{2}\vec{a}t^2}{t} = \vec{v}_i + \tfrac{1}{2}\vec{a}t = \tfrac{1}{2}(2\vec{v}_i + \vec{a}t) = \frac{\vec{v}_i + (\vec{v}_i + \vec{a}t)}{2} = \frac{\vec{v}_i + \vec{v}}{2} . \quad (2)$$

Note this well: <u>these four relationships are nothing more than combinations of the definitions of velocity and acceleration</u>, for the special case of constant acceleration.

Now, we have four *kinematic equations* that are valid in the special case of constant acceleration. Various combinations and perturbations of these should allow for solving most problems. Here, however, is a warning: do not rely on the equations by themselves to solve problems. The equations are in a sense tools, but it still requires the brain to direct their use.

We did the derivations for three dimensions, but this section deals with one dimensional motion, so let's change our notation a bit:

$$v = v_i + at \quad (1)$$

$$v_{AVE} = \frac{v_i + v}{2} \quad (2)$$

$$x = x_i + v_i\, t + \tfrac{1}{2} a\, t^2 \quad (3)$$

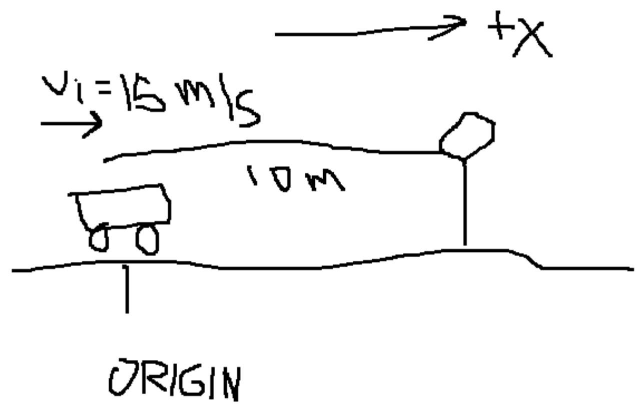
$$v^2 = v_i^2 + 2\, a\, \Delta x \quad (4)$$

Notice that the dot product was dropped from the last equation. This will still work out if we use the following convention. Since the displacement, velocity, and acceleration are indeed all vectors, we need a mechanism to describe their directions. So, let's say that if the velocity is to the positive x direction, we'll insert a positive value in for v in the equations, and if the velocity is pointed in the negative x directions, we'll insert a negative number. Same for the acceleration and displacement. Since we're now operating in one dimension, two non-zero vectors can only be either parallel or anti-parallel. Now, if the acceleration and displacement are in the same direction, whether both positive or negative, the dot product would give us a positive result. Contrarily, if they were in opposite directions, we'd obtain a negative value. This convention maintains those results. (+)(+) = (-)(-) = (+) and (+)(-) = (-)(+) = (-).

OK, now, let's work through some 1-d examples to help firm up your understanding.

EXAMPLE 2-4

A distracted driver traveling at 15 m/s notices a stop sign when he is 10m from the stop line. If the car decelerates at 6 m/s^2, how quickly is the car moving as it passes the stop line?



The first step is to sketch a diagram; this helps to visualize the situation and explain to any readers of your solution how you solved the problem.

We'll label the origin and indicate which direction is +x. Then, we'll decide where (or when) the problem starts and ends; in this case, the problem starts when the car is 10 me from the sign, and ends when the car is at the sign. Next, we'll make an inventory of the quantities we know either implicitly or explicitly, as well as what we want to figure out:

x_i	0 m
x_f	+10 m
v_i	+15 m/s
v_f	? ←
a	-6 m/s^2
t	?

Notice that the acceleration is listed as -6 m/s^2. We assume this based on the choice of the word 'decelerate.' This implies that the car is slowing, and in one dimension, a slowing object's acceleration will be opposite to its velocity. Likewise, if speeding up, the acceleration and velocity will be in the same direction.

Now, as much as we preach that physics is not just plugging numbers into equations, that's what we're going to do at the beginning. Don't worry, the class will get harder. What we would really prefer is if there is one kinematic equation that has only quantities we know and what we want to know, and nothing else. Sometimes, we must use one relationship to find an intermediate quantity, then use another relationship to solve the problem. In this example, we're good to go with Eq. 4.

Indicate which relationship you are using, and show a reasonable number of steps to the answer. It is best if you can solve the problem in symbols, putting in actual values at the end. This is for two reasons: first, manipulating the relationships will help you see the relations among the various quantities and, more practically, you won't have to solve the entire problem again if your boss comes by and tells you the initial velocity was actually 20 m/s. There are some exceptions, of course.

Eq.4

$$v^2 = v_i^2 + 2\,a\,\Delta x$$

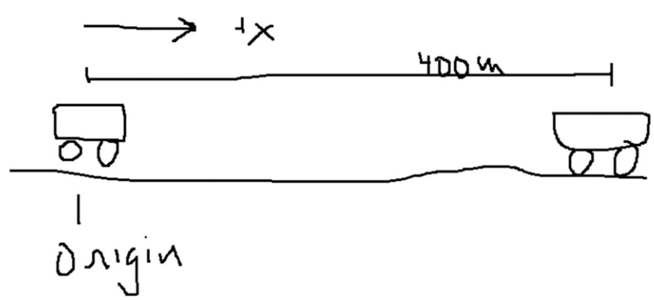
$$v = \sqrt{v_i^2 + 2\,a\,\Delta x} = \sqrt{15^2 + 2\,(-6)\,10} = \pm\,10.2$$

We need to be careful with square roots. We know that the car is heading in the positive x direction, so

$$v_f = +10.2\ \mathrm{m/s}$$

EXAMPLE 2-5

A car with a velocity of 30 m/s accelerates at 4 m/s2. How long will it take for the car to cover 400 m?



In this case, the diagram is probably not so critical. However, the inventory is.

x_i	0 m
x_f	+400 m
v_i	+30 m/s
v_f	?
a	+4 m/s^2
t	? ←

Let's try Eq. 3:

$$x = x_i + v_i\,t + \tfrac{1}{2}\,a\,t^2$$

Now, here we will need to solve a quadratic equation, and so this is a situation when putting in numbers early is probably warranted.

$$400 = 0 + 30t + \tfrac{1}{2}\,4\,t^2$$

Put this into the 'standard' form:

$$2\,t^2 + 30t - 400 = 0$$

Then

$$t = \frac{-30 \pm \sqrt{30^2 - 4(2)(-400)}}{2(2)} = +8.5\,\text{sec OR} - 23.5\,\text{sec}$$

And now, we must exercise some judgement. Since the car travels the4 00 meters after the problem began at t = 0, the positive answer is the correct one:

t = +8.5 sec.

We might ask ourselves, though, is there any physical significance to the other solution. Well, here's the thing. These equations are stupid, they're just tools. You are the craftsmen who use the tools. The equation we wrote above makes the assumption that the car has an acceleration of +4 m/s^2, and that it has always had an acceleration of +4 m/s^2. The equation 'thinks' that the car started off at positive infinity, an eternity ago, moving in the negative x direction. Over time, it slowed until it passed our finish line (moving the wrong way) at t = -23.5 seconds. It continued to slow, stopped, and then began moving in the positive direction, arriving at the origin at t = 0 seconds, and at the finish line at t = +8.5 seconds. It will then continue to accelerate back towards infinity for the rest of eternity.

HOMEWORK 2-2

Consider a car moving at 40 m/s when its brakes are applied to slow it to 18 m/s (speed camera!) within a distance of 20 meters. What is the car's acceleration? How much time does it take to reach the location of the camera?

Mastery Question

A train moving at 15 m/s passes the origin ($x_i = 0$) at $t_i = 0$. At that instant, the engineer hits the brakes, giving the train an acceleration of - 0.5 m/s^2, so that it comes to a stop. Where is the train after 40 seconds? Click here for the solution.

DISCUSSION

What should we do when the acceleration is not constant?

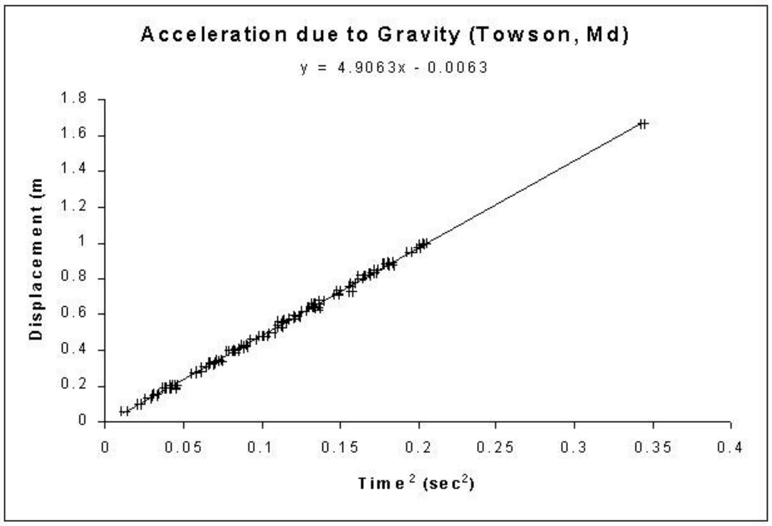
So long as it is constant over intervals and changes abruptly, we can treat each individual interval as above, using the final values of the quantities in one interval as the initial values for the next interval. Otherwise, it's a calculus problem.

HOMEWORK 2-3

A race car accelerates from rest at 9 m/s^2 for 400 m along a straight track, then decelerates to a stop over an additional distance of 1200 m. What was the car's maximum speed and how much time did this process take?

Acceleration due to Gravity

In the very special case of an object moving freely near the surface of the earth under no other influence except the earth's gravity, the acceleration of the object will be (approximately) 9.8 m/s^2 downward. You will verify this in a laboratory exercise. Your text probably refers to this quantity as the *acceleration due to gravity*, *g*. I would prefer that you use the symbol a_g, reserving g for the *gravitational field strength*, which we shall discuss in Section 5.



The results of an experiment by the Fall 2003 PHY542 class are shown here. After dropping a mass from rest through vertical displacement h and measuring the travel time, the data were plotted as h *vs* t^2. If the kinematic relationships are true, the slope of this curve represents half of the acceleration due to gravity, a_g. A value of 9.81 m/s^2 is within about 0.14% of the accepted value in Towson.

EXAMPLE 2-6

A ball is thrown from the street such that it rises past a 25m high window ledge at 12 m/s. Find a) the velocity with which it was launched, b) the maximum altitude above the street it reaches, c) how long ago it was thrown, and d) the time until it returns to the ground.

This is actually several problems, because there are several starting and several ending points. Let's address Parts a and c first, where the starting point is on the ground and the ending point is 25 meters up. Set the origin at ground level and let up be positive.

x_i	0 m
x_f	+25 m
v_i	? ←
v_f	+12 m/s
a	-10 m/s^2
t	? ←

KEq 4

$$v^2 = v_i^2 + 2\,a\,\Delta x$$

$$v_i = \sqrt{v_f^2 - 2\,a\,\Delta x} = \sqrt{12^2 - 2\,(-10)\,25} = \pm\,25.4\,\frac{m}{s}.$$

Since the ball was thrown upward, we want to positive root: +25.4 m/s.

KEq 1

$$v = v_i + at \quad \rightarrow \quad t = \frac{v - v_i}{a} = \frac{12 - 25.4}{-10} = 1.35\text{ sec}\ .$$

Part b is really a different problem. We can make use of the fact that the ball stops for a moment at the top of its trip.

x_i	0 m
x_f	? ←
v_i	+25.4 m/s
v_f	0 m/s
a	-10 m/s^2
t	?

KEq 4

$$v^2 = v_i^2 + 2\,a\,(x - x_i)$$

$$x = x_i + \frac{v^2 - v_i^2}{2\,a} = 0 + \frac{0v^2 - 25.4^2}{2\,(-10)} = 32.3\text{ m}\ .$$

Part d is yet a different question. I'll assume to means, 'How long from passing the window on the way up until it hits the ground?'

x_i	25 m
x_f	0 m
v_i	+12 m/s
v_f	?
a	-10 m/s^2
t	? ←

KEq 3

$$x = x_i + v_i\,t + \tfrac{1}{2}\,a\,t^2$$

This will be a quadratic equation, so let's insert the values now.

$$0 = 25 + \ 12t + \tfrac{1}{2}(-10)\, t^2$$

Re-arrange into the standard form for solving:

$$5\, t^2 - 12t - 25 = 0$$

$$t = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(5)(-25)}}{2(5)} = \ +3.74 \text{ sec OR} - 1.34 \text{ sec.}$$

We, of course, want the answer in the future, so t = 3.784 seconds.

DISCUSSION 2-4

Speaking of that highest point, what is the acceleration at that point? It is common to assume that it is zero, but that confuses that velocity with the acceleration. We can look at the graph above and see that the slope of the v(t) graph when v = 0 is still -9.8 m/s^2. Or, think opf the velocity just before the peak (positive) and just after the peak (negative); the acceleration measures the change in velocity, which became more negative in that time interval. Or think of it this way, since acceleration is related to the change in velocity, if a_g were zero at the peak, what would the object do? No acceleration implies no change in velocity, so the object would just hang in space, an event counter to our experience.

HOMEWORK 2-4

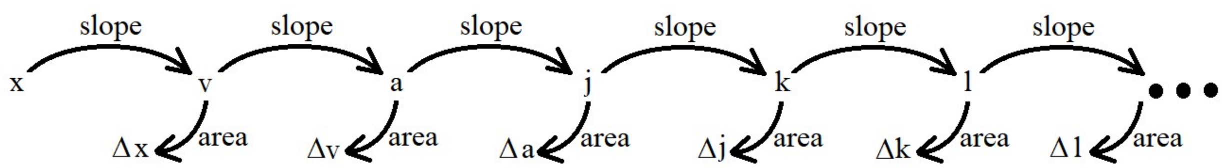
A missile is launched upward from the ground with an acceleration of 40 m/s^2. The burn lasts for 5 seconds. After that, the rocket coasts upward before stopping and falling back to earth. What maximum altitude above the earth does the missile reach? How much time was required to arrive there?

A Different Graphical Interpretation

One last topic. We saw that the acceleration can be found from the slope of a line tangent to a velocity *v.* time curve, that is, from the derivative. Can we get any other information from such a curve? Of course!

$$v(t) = \frac{dx}{dt} \quad \rightarrow \quad dx = v(t)\, dt \quad \rightarrow \quad \Delta x = \int v(t)\, dt \quad ,$$

That is, the displacement is the <u>area</u> under the velocity *v.* time curve, or the integral. The slope of the line tangent to the acceleration curve is the jerk. What does the area under this curve represent?



Note One

So, just in case you're unhappy with $\mathbf{v} \cdot d\mathbf{v} = v\, dv$, here is a quick, explicit demonstration. Suppose we were to do this in one dimension only. Then, $\mathbf{v} \cdot d\mathbf{v} = v_x\, dv_x$ with the sign of v_x accounting for the cosine term of the dot product. On integration, this becomes $^1/_2\, v_x^2$. We can of course write this in one dimension for the y and z directions as well, and the corresponding integrations result in $^1/_2\, v_y^2$ and $^1/_2\, v_z^2$.
Now,
$\mathbf{v} = v_x\boldsymbol{i} + v_y\boldsymbol{j} + v_z\boldsymbol{k}$, and $d\mathbf{v} = dv_x\boldsymbol{i} + dv_y\boldsymbol{j} + dv_z\boldsymbol{k}$.
So,
$\mathbf{v} \cdot d\mathbf{v} = (v_x\boldsymbol{i} + v_y\boldsymbol{j} + v_z\boldsymbol{k}) \cdot (dv_x\boldsymbol{i} + dv_y\boldsymbol{j} + dv_z\boldsymbol{k}) = v_x\, dv_x + v_y\, dv_y + v_z\, dv_z$.

This integrates to $^1/_2\, v_x^2 + {}^1/_2\, v_y^2 + {}^1/_2\, v_z^2$, which becomes, with the help of the Pythagorean theorem, $^1/_2\,(v_x^2 + v_y^2 + v_z^2) = {}^1/_2\, v^2$.

In fairness, this works only if you intend to integrate!

EXERCISE 2-1 Solution

In the first case, since the trip is made without reversing direction, the distance will be the same as the magnitude of the displacement, or 2 meters. Or, if you prefer, we've taken 2 m worth of steps. In the second case, each segment of the trip is one way, so we can count segment by segment.
3→15 12 m
15→ -8 23 m
-8→ 5 13 m
48 meters in total