SECTION 5 – THE FIRST PICTURE

Students often get irritated with me when I point out that, up to this moment, we've really not done any physics at all. We did discuss the acceleration of objects due to gravity near the earth's surface, but the rest was really just definitions. Kinematics is the descriptive study of the motions of objects, but we'd like to know why objects move the way they do. To understand this, at least to the degree allowed in our one semester course, we'll be looking at three 'pictures,' or ways of looking at problems. To many people, these three pictures seem separate and unconnected, but in fact, they are all really the same, just twisted around a bit. Each is particularly well suited to addressing certain types of problems; the other two can of course be used as well, but not as efficiently.

In the introduction, I mentioned that the course is structured much like a traditional two-column proof Geometry course. In this section, we will introduce the two axioms on which the pictures are built. In Physics, we call these axioms *laws*. What exactly is a law? Like the axions of Geometry, laws are ideas that we observe never to be false and which we then assume are true. The purpose of experimentation is to try to show that these laws are false; the more unsuccessful we are at that, the more confidence we have that they are true. Of course, sometimes, we fool ourselves into thinking that something is true, then find out that it was actually a special case, or that our experiments just weren't accurate enough.[1] The notions many students have when they start a course in classical mechanics are often referred to as *Aristotelian*; part of the purpose of these classes is to disabuse students of these Aristotelian notions.

THE LAWS OF MOTION

Dynamics is the study of why objects move the way they do, particularly with regard to *forces*. The root of the word is Greek for force.[2]

DISCUSSION 5-1

> How would you define a force? What definition were you given in grade school? Is that good enough? Is force a scalar or a vector? Is there a difference between pushing something to the left and pushing it to the right?

DISCUSSION 5-2

> Let's start by making some observations. We'll place a book on the table. Watch the book carefully. What is the book doing? We often use the expression 'at rest' to describe this situation. What will it be doing an hour from now? What about next semester? What about when I finally retire? If we want to book to be not at rest, what must happen?

[1] An excellent example of this is that some of the ancients believed that the earth revolves around the sun. They reasoned that, if this were true, an effect known as *stellar parallax* would be observed. The effect was in fact not observed (it was too small to be detected with the techniques available at the time) until two millennia later.

[2] Indeed, a *dyne* is a unit of force, one we won't be using.

Suppose I next toss the book onto the table. What phrase might we use to describe the book? Once the book hits the table, what happens, and why? What stopped the book?

What is necessary to change the motion of an object? If we say that the 'motion is constant,' which quantity from Section 2 are we really referring to? Which quantity from Section 2 measures the change in the motion?

CHEESY EXPERIMENT 5-1

Let's do a quick experiment to see how this works. We've decided that a force is necessary to change an object's motion or velocity, that is, it causes acceleration. I have a cart on wheels (to minimize the effect of friction, whatever that is) and a force-o-meter marked off in some weird units. I have some confidence that the force-o-meter works, in that as I pull harder, the numbers increase on the dial. As I pull on the cart, you can hear from the sound of the wheels that it's accelerating. If I pull with twice as much force, the acceleration is higher. Briefly, as $F\uparrow$, $a\uparrow$.

DISCUSSION 5-3

We should try to be a bit more explicit in the relationship between acceleration and force. If no force means no acceleration, and more force means more acceleration, what is the simplest relationship between them that you can think of? What about the directions of the force and acceleration?

If we assume that, perhaps, the universe behaves as simply as possible, we might conjecture that the acceleration is proportional to the force. Then, we make a hypothesis that $\vec{a} \sim \vec{F}$. We may be wrong, of course. It may turn out that the acceleration is proportional to F^2 or F^3. Maybe the force and acceleration are actually not in the same direction.[3] At some point, we'll do this experiment much more carefully and find out.

What else can we change? We don't want to base our notions on just one experiment; the results may have been coïncidental. Best to vary as many parameters as possible.

CHEESY EXPERIEMENT 5-2

Let's repeat by keeping the force constant and doubling the mass of the cart. What do you notice? Is the acceleration larger or smaller when the mass is increased?

DISCUSSION 5-4

What relationship would you say exists between the acceleration and the mass of an object?

We might write that as $m\uparrow$, $a\downarrow$. Note something interesting. In chemistry, you're told that the mass is a measure of how much material there is in an object. Here, we see that the mass of an

[3] Just wait for Physics III!

object is a measure of how hard it is to accelerate that object. The simplest relationship between the two would be that the acceleration and the mass are *inversely proportional*: a ~ 1/m. Of course, we may be wrong, but we at least have a hypothesis.

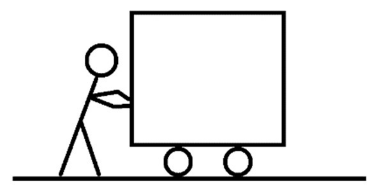
We've performed some simple experiments and conjectured that the acceleration is proportional to the applied force if the mass is held constant, and inversely proportional to the mass of the force is constant. Let's synthesize these ideas into one:

$$\vec{a} \sim \frac{\vec{F}}{m} \,.$$

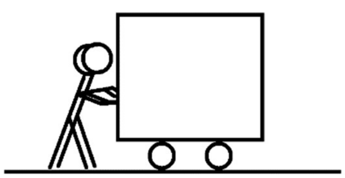
If we choose the correct units, we can make the proportionality an equality. Let the force necessary to accelerate one kilogram at one meter/second^2 be called one *newton*. Then,

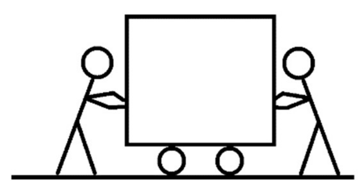
$$\vec{a} = \frac{\vec{F}}{m} \,.$$

DISCUSSION 5-5



What if there is more than one force acting on an object? Consider poor Joe, who has to push a crate across the warehouse floor by applying a force F to the right. The next day, Joe gets his twin brother Jeb to help. How much force is applied to the crate by the boys? What would you expect the acceleration of the crate to be today, compared to that of yesterday? On the next day, Jeb isn't available, so Joe asks his other twin brother Jake to help out. Jake, however, seems to never quite 'get it.' How much force is applied to the crate in this situation? What would you expect the crate's acceleration to be on this day? So, what must we do when there is more than one force?





Putting all of these ideas together gives us the s*econd law of motion*:

$$\vec{a} = \frac{\sum_n \vec{F}_n}{m} \,.$$

Note that when we sum the forces,[4] we include only the forces that are actually acting on the object of interest. Other forces influence the motions of <u>other</u> objects. I'll show you a way of keeping track of which forces act of which objects.

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[4] You may not be familiar with this notation. $\sum_n \vec{F}_n$ means simply to add up all the forces with n as a counting number: $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots$. Remember to add the forces as vectors.

Lastly, although the form of the second law given above is conceptually the better, we are going to re-arrange it so that it is more convenient to use for problem solving:

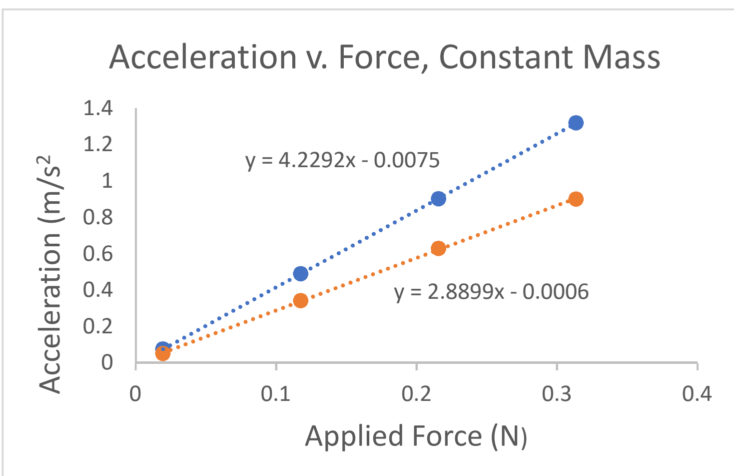
$$\sum_n \vec{F}_n = m\vec{a} \ .$$

The reason I like this is that all the effort of solving a problem is done on the left side, and the right side is always mass times acceleration. Let me emphasize that $m\vec{a}$ is not a force. Forces are the cause, and acceleration is the result.

You may ask, 'What's the *first law of motion*?' Well, it's a special case of the second. If there are no forces acting on an object, its acceleration is zero and the velocity will be constant. If the object is at rest, it will remain at rest, and if it's moving, it will move with constant velocity.

EXPERIMENT 5-3

We placed a mass, which we call a glider, on a horizontal air track. The track acts much like an air hockey table. There are small holes through which air is forced to lift the glider off the track surface to minimize friction (whatever that is). A force was applied to the mass and the resulting accelerations were measured. Here are the results, plotted in two ways. In the first graph, each line represents runs with a constant mass; the fact that the lines pass through the origin (well, the intercepts are very small compared to the values plotted) indicates that the respective accelerations and applied forces are proportional. What's more, matching the equation for a line to our hypothesized relationship leads us to predict that the slope should be the inverse of the mass:



$$y = (\text{slope})x + \text{intercept}$$

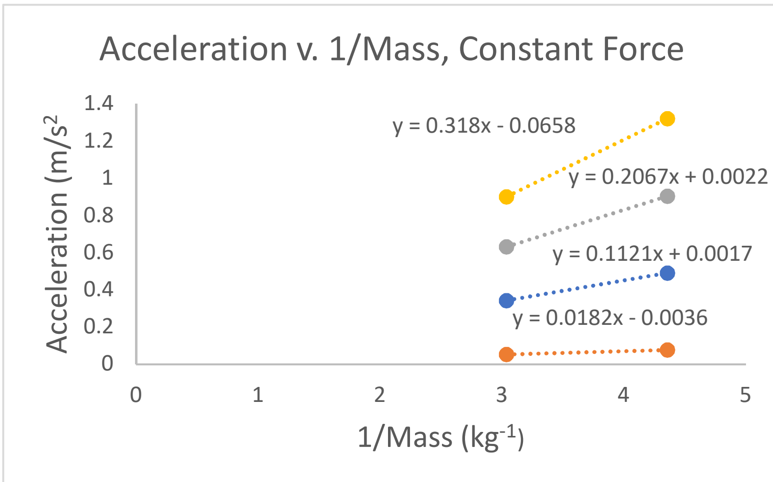
$$a = \left(\frac{1}{m}\right)F + 0 \ .$$

Actual Mass	Mass from graph	*Per cent* difference
0.2295 kg	0.2365 kg	+ 3.1 %
0.3293 kg	0.3460 kg	+5.1 %

To gain a bit more confidence, let's plot these data differently, acceleration *v*. inverse mass. In this case, we're trying to fit the data to this relationship:

$$a = (F)\frac{1}{m} + 0\,.$$

Once again, we should see lines passing through the origin with the slopes equal to the respective applied forces.



We can see that the intercepts are quite small, compared to the smallest acceleration values, so the proportionality condition seems to be met. Let's look at the forces.

Actual force applied	Force from graph	*Per cent* difference
0.3137 N	0.3180 N	+ 1.4 %
0.2156 N	0.2067 N	- 4.1 %
0.1176 N	0.1121 N	- 4.7%
0.0196 N	0.0182 N	- 7.1 %

Without doing an uncertainty analysis, we can't really determine if our hypothesis is justified, but I think that, perhaps, we may have some confidence that it is correct, subject to future, more careful validation.

HOMEWORK 5-1

A net force of 45 N is applied to a mass of 16 kg. What will be the mass's acceleration? How much force should be applied to a 27 kg mass to give it the same acceleration?

The *third law of motion* seems to be the one students have the most trouble with, although it really is the easiest to understand: If object A exerts a force on object B, then B exerts a force on A that is of the same type, equal in magnitude, and opposite in direction. Mathematically, we write that

$$\vec{F}_{B,A} = -\vec{F}_{A,B}\ .$$

Think about this scenario: A speeding car A rear-ends a parked car B at a red light; the parked car B is accelerated forward because of the force exerted by A, while A slows down due to the force exerted backwards on it by B. Two forces that fulfill this description are referred to as a *third law* *pair*. To be a third law pair, the forces must fit the description given above, *e.g*., A pushes B and B pushes on A.

CHEESEY EXPERIMENT 5-4

Let's do a quick check of this concept with two *force-o-meters*.

Third Law.MOV

We see that the forces each exerts on the other are equal in magnitude. Later in Section 7, we'll show some experimental results that will help grow confidence in the third law.

DISCUSSION 5-6

Our book is still sitting on the table. There is a gravitational force exerted on the book by the earth (this is called the book's *weight*) and a *force of contact* acting upward on the book from the table. If the acceleration of the book is zero (it's not moving), then what can we say about the two forces just mentioned? Do these two forces form a third law pair? If not, what are the other halves of each pair? Did you give them the A on B, B on A test?

If there's doubt on what constitutes a third law pair, just change the subjective and objective parts of the sentence around. It's not too hard to believe that if the table pushes up on the book, then the book pushes down on the table. Harder perhaps to believe that if the earth pulls down on the book, the book pulls up equally on the earth. Indeed, if I were to drop a book, the earth would accelerate upward to meet the book!

DISCUSSION 5-7

Why don't we notice the earth moving upward toward the book?

Forces that form third law pairs are often called action-reaction forces. I don't like this terminology, because it gives the impression that one force occurs first, then the other. Third law pair forces occur simultaneously (in this semester).

HOMEWORK 5-2

A positively charged proton (mass = 1 dalton)[5] repels a positively charged alpha particle (mass = 4 daltons) with a force of 0.5 pico-newtons. What force does the alpha particle exert on the proton?

TYPES OF FORCES

[5] More or less. A dalton is 1/12 the mass of a carbon 12 atom. A pico-newton is one quadrillionth of a newton.

In this course, you will encounter several types of forces. We'll start with three of them, then add in the others when we're ready.

Weight

We mentioned above that there is a force on objects that are near the surface of the earth that is associated with gravity, which we shall call the weight, $\vec{W}$. When an object is in free fall, the only force acting on it is $\vec{W}$, which we know from lab causes the object to experience an acceleration of a_g downward. Using the second law, we can write that

$$\vec{W} = m\,\vec{a}_g \ .$$

Since all objects will fall with this same acceleration, the force necessary to give an object a certain acceleration is proportional to the mass of the object by some factor which we'll call $\vec{g}$, the *gravitational field strength*:

$$\vec{W} = \vec{g}m \ .$$

Note that $\vec{g}$ must be a vector quantity (pointing downward). What is the value of g? The fact that

$$\vec{W} = \vec{g}m = m\,\vec{a}_g \quad \rightarrow \quad \vec{g} = \vec{a}_g \ .$$

However, $\vec{g}$ and $\vec{a}_g$ are two different quantities; they have the same value, the same dimension, and the same direction, but different units. Since g is the gravitational force *per* unit mass the earth exerts on an object near its surface, $\vec{g}$ is 9.8 newtons/kg, downward.[6]

DISCUSSION 5-8

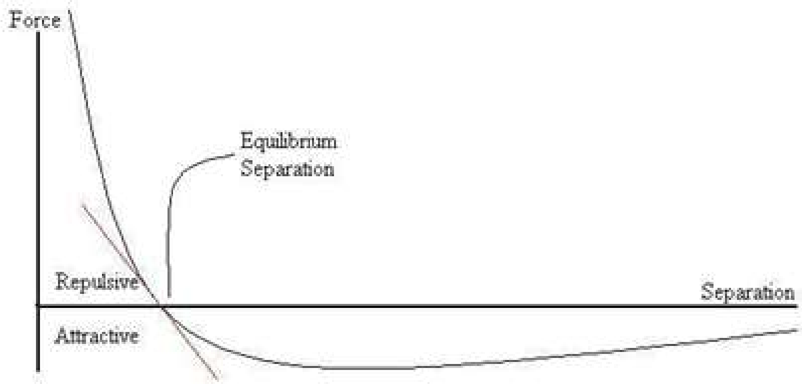
> Drop a ball. As it falls, is there an acceleration? Is it specifically a_g? Is there a gravitational field? Now, make the ball smaller and drop it. Is there any acceleration? Is there a gravitational field? How strong is it? Keep making the ball smaller and smaller until there is no ball left at all. Is there an acceleration? Is there a gravitational field? How strong is it?
>
> Next, place the ball on a table. Is there an acceleration? Is there a gravitational field? How strong is it?

Now, as we did for a_g, we will round off the value of g to 10 N/kg for the purposes of homework and exams. In lab, we will be more careful.

(Normal) Force of Contact

[6] This may seem like a big deal over nothing, but there is an analogous situation in Physics 2 that generally gives students a hard time. Better to start thinking this way now.

Another type of force is the *contact force*, which is due to the fact that two objects are actually touching one another. We will be considering two different contact forces in this course, but we'll start with the one that is perpendicular (normal) to the surface of contact. The nature of this type of force can be thought of as being due to the electronic bonds between atoms or molecules in each of the materials. You may have learned in chemistry that the forces between these particles looks a bit like this as a function of the separation:



The forces of repulsion and attraction cancel at the equilibrium separation (F = 0). Around this point, it can be shown that the system acts much like balls connected by springs. So, as the two macroscopic objects come into contact, the 'springs' are compressed and produce forces which act to push the objects back apart. Or the objects could be glued together and then pulled apart, so that the 'springs' stretch and try to pull the objects back together. In either case, the forces are due to contact between the objects and are directed perpendicularly to the interface between the objects.

As an example, suppose you arrive home and set your bookbag on your couch. At first, the bag will move downward into contact with the couch. As the bag pushes into the cushion, it compresses the springs there, which, as we'll see later, start to push back upward. In the end, the bag comes to rest with the springs under it compressed. If we think of the atoms in an object as balls connected by springs (instead of electric bonds) we can imaging the same thing happening in microcosm.

Tension

Often, we speak of the *tension* in a string or rope. We'll define the tension to be the force the string exerts on the object it's attached to. In this course, we usually assume that the strings are massless and inextensible (they don't stretch).

Let's make an argument that the tension at each end of such a string is the same as at the other end (except of course opposite in direction). Consider a rope used in a tug of war game. The team on the right pulls to the right with force F_R, and by the third law of motion, the string exerts the same magnitude force (the tension T_R at that end, by our definition) on the team. Likewise, the team on the left exerts a force F_L on the rope, and the other half of that third law pair is the tension T_L on the left end of the rope. If the rope is massless, any difference in applied net force would cause an infinite acceleration. Hence, $F_R = F_L$ and $T_R = T_L$.

Strings and ropes are often looped over wheels. This does nothing more than change the direction of the tensions at the ends, so long as the wheel is itself frictionless and massless. Situations where the wheel is not frictionless or massless will be treated later in the course.

APPLICATIONS OF THE LAWS OF MOTION

First, several comments on notation. In kinematics, we used the sign of a vector's value to indicate the direction of the vector. If an object moved in the +x directions at 3 m/s, we said that $v_x = +3$. If it moved in the -x direction, we said $v_x = -3$. We're going to stick with that notation for kinematic quantities here, but we will do something different with forces. We will always write the magnitude of the force, but insert the correct sign in front of the force to indicate the direction. In particular, we will always use positive 10 N/kg as the value for g regardless of whether up or down is positive. Second, although we were very careful in past sections to measure the angle for finding components CCW from the x axis, we shall abandon that approach at this point. Now, we will just make use of the most convenient angle and take care of the signs as described above.

DISCUSSION 5-9

> Consider a person standing on a spring scale. What does the scale actually measure? Suppose he holds the scale up against the wall and pushes on it horizontally. Does the scale measure the man's weight? How does a scale indicate a person's weight?

This is one of the first things we need to be able to do, decide what forces act on an object. The man's weight does <u>not</u> act on the scale, the man's weight is the force of (gravitational) attraction between the <u>earth</u> and the man, and that force acts <u>on the man</u>. The man and the scale are in contact with one another, and it is the normal force of contact that the scale measures. For example, if I were to place the scale on the wall and lean against it, the scale would not be measuring my weight. To keep track of the forces acting on an object, we can use a *free body diagram*, which is just an accounting tool to isolate each object for analysis. Draw each of the forces with its tail at the center of the body under consideration (here, the man). Here we see the weight (force of gravity of the earth acting on the man) and the normal force (force of contact of the scale acting on the man). What about the force of the man acting on the scale? Well, that's a force on the scale, not on the man; that force would go on the free body diagram of the scale..

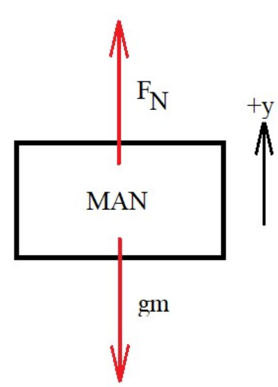
EXAMPLE 5-1

If the man is not moving, his acceleration is zero, and so we can write, using the second law and making upward positive, that

$$\sum_n \vec{F}_n = m\vec{a}$$

$$+F_N - gm = 0$$

$$F_N = gm \ .$$



Since, by the third law, the normal force on the man from the scale is the same magnitude as that of the normal force of the scale on the man, the reading on the scale is numerically equal to the weight of the man, but it is not the weight of the man.

Now, let's put the man and the scale in an elevator that is accelerating upward. The diagram is similar to the one above. Writing the second law results in:

$$\sum_{n} \vec{F}_n = m\vec{a}$$

$$+F_N - gm = ma$$

$$+F_N = ma + gm$$

So, we see that if the elevator is accelerating upward, the scale reading will be higher than the man's weight, while it will be lower if the elevator's acceleration is downward.

DISCUSSION 5-10

While you may not be gnurdy enough to ride in an elevator with a scale, you probably have noticed this effect. There are sensors in your body that can tell you when one layer of you is compressed against another layer of you. How do you 'feel' when your elevator starts to move upward? Do you feel as if you are heavier? If you were standing on a scale, what would it read? What about when your elevator starts to descend?

What would the scale read if the cable were cut and the elevator car went into freefall and what would the person be doing inside the car? In such a case, would the person be weightless?

Are astronauts on the ISS weightless? If they were weightless, what would happen to them? Astronauts in training ride the 'Vomit Comet.' Bing it. Commercial versions are available.

HOMEWORK 5-3

An 90 kg man stands in an elevator. What force does the floor of the elevator exert on the man if

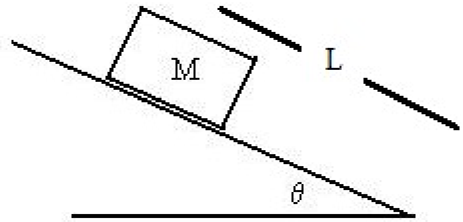
a) the elevator is stationary?
b) the elevator accelerates upward at 1.2 m/s^2?
c) the elevator rises with constant velocity 3 m/s?
d) while rising, the elevator decelerates at 0.5 m/s^2?
e) the elevator descends with constant velocity of 2.5 m/s?

HOMEWORK 5-4

A softball (mass = 0.19 kg) is thrown directly upward so that it leaves the pitcher's hand at 4.5 m/s. The pitcher's hand moved through 1.5 meters as he threw the ball. What force did the pitcher exert on the ball?

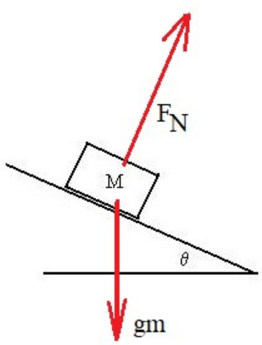
EXAMPLE 5-2

Here is a problem we shall use as the model for presenting solutions. We will be revisiting it from time to time.



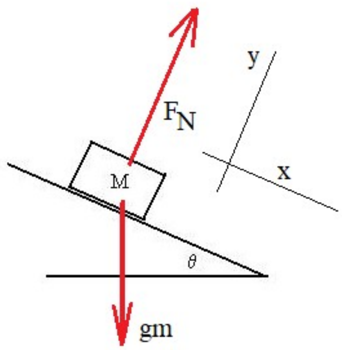
Consider a block of mass M on a frictionless plane inclined at an angle θ from the horizonal. If the block is released from rest from a point a distance L up the incline, how quickly will it be moving when it reaches the bottom? Let theta be 37º, M be 5 kg, and L be 2 meters.

Next, we can analyze the forces acting on the mass. We can do an inventory, much as we did for kinematics. Is there a weight? Sure, and remember that there is only one weight *per* object. Is the mass touching something? It's touching only the incline, so there is one normal force of contact. Are there any strings or rope or the like? No. The free body diagram for the mass is then:



Notice that the normal force of contact is perpendicular to the surface. A common misconception is that the normal force points upward, and is probably due to the fact that students start by trying problems like the elevator situation above.

Next, we'll pick a coördinate system. There is a need for some experience here, but here is a hint: we certainly expect the block to accelerate along the plane, and not to either jump off the plane or burrow into it. You may remember Rule One from Section 3: choose a system such that the acceleration is along one of the axes. It will be much easier to solve this problem if we orient the axes parallel and perpendicular to the plane. It's not impossible to solve the problem otherwise, but it's a lot tougher mathematically. If the problem is such that the acceleration is zero, then this aspect is not so important and other considerations can be examined.

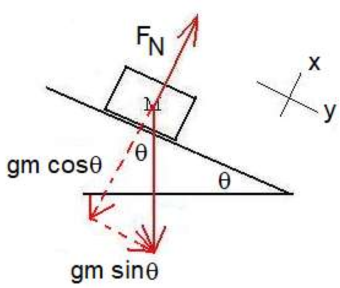


Now, we can write the second law as

$$\vec{F}_N + \vec{g}m = \vec{a} \ ,$$

which is not very useful, since we then have one equation with two unknowns. However, we decided back in Section 1 that if two vectors are equal, then their components must be independently equal. So we'll break this equation into two separate equations, one for x and one for y.

First, the components. It would probably be a good idea to review. We've decided to make the coördinate system as shown in the figure, based on the object's presumed acceleration direction. F_N is already completely in the +y direction, but gm is mixed. We must replace gm with two other vectors that, when added together, equal the original vector, with one parallel to the x-axis and the other parallel to the y-axis. Note that we are <u>not</u> using the angle as measured from the +x axis to find the components, but instead are simply making use of the trig identities with the angle we are given in the triangle. We will assign the proper sign to the directions of these forces when we write the second law equations. Since the x-component of the weight points in the positive x-direction, we'll place a plus sign in front of it. Similarly, the normal force is in the +y-directions, but the y-component of the weight in the negative y-direction.



$$\text{x:} \quad + gm\sin(\theta) = ma_x$$

$$\text{y:} \quad + \ F_N - gm\cos(\theta) = ma_y = 0$$

In this case, the y equation is not useful, but the x equation tells us that

$$a_x = g\sin(\theta) \ .$$

Keep in mind that in our method, g is always positive; we took care of the direction by placing the correct sign in front of the terms in the second law equations.

DISCUSSION 5-11

When we finish with a major section of a solution, we should ask if the result makes some sense. If theta were zero, what would the acceleration be? If theta were ninety degrees, what would the acceleration be? Do those values make sense?

EXAMPLE 5-2 Continued

So, for the values given in the problem, the acceleration will be

$$a_x = g\sin(\theta) = 10\sin(37^o) = \ 10(0.6) = 6 \text{ m}/s^2 \, .$$

Now that we have the acceleration (and it's constant!), we will choose a kinematic equation to find the final speed. Let's place the origin at the starting location.

$x_i = 0$ (starts from the origin)
$x_f = 2m$
$v_{xi} = 0$ (starts from rest)
$v_{xf} = ?$ ←
$a_x = +6$ m/s2
$t = ?$

KEq 4 seems like a good choice.

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

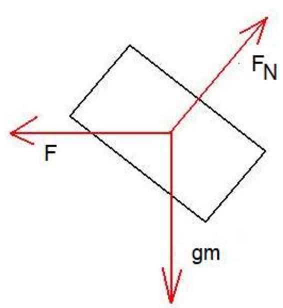
$$v_{xf} = \sqrt{v_{xi}^2 + 2a_x(x_f - x_i)} = \sqrt{0^2 + 2(6)(2-0)} = +\,4.9\ \mathrm{m/s}\,.$$

We take the positive root because the box is sliding in the +x direction.

We will be returning to this same problem a number of times, using it as a familiar point of reference for later discussions.

In the previous example, we alluded to 'other considerations' in choosing a coördinate system. As stated, it's generally best to align the axes so that the acceleration is along one of them. If the acceleration is zero, this is not applicable, and a judicious choice of axes might reduce the amount of math we need to do. This is Rule Number Two. Consider this problem:

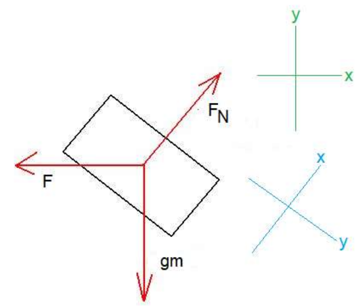
EXAMPLE 5-3

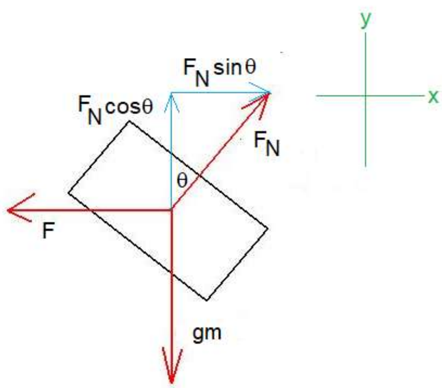


Apply a force F horizontally to the block of the previous problem so that the block remains stationary. Find F and the normal force, F_N.

Let's start with a free body diagram. It bears repeating that the normal force is perpendicular to the surface between the box and the incline. Also, since the object is motionless, there is no acceleration and Rule Number One is moot.

Now, if you were to set up the tilted, blue coördinate system as in the previous example, you might notice that you would have to decompose two vectors. However, if you chose the green system, only one vector would require decomposition. That may make the algebra easier in that there would be fewer terms to manipulate (four *vs* five). Let's go with the green system and find the components of F_N. A small amount of geometry shows that if the ramp is inclined at angle theta, then the normal force is inclined theta from the vertical.





Now we write the second law for x and y:

$$\text{x:}\quad +\ F_N \sin(\theta) - F = ma_x = 0$$

$$\text{y:}\quad +\ F_N \cos(\theta) - gm = ma_y = 0\ .$$

At this point, it is worth making a general comment. Note that the acceleration terms are zero. This is because this particular object is stationary. This is not always the case. A common error is to write a = 0 for every problem.

Here's a neat mathematical trick. We're going to divide these two equations. First, some re-arrangement.

$$F_N \sin(\theta) = F$$

$$F_N \cos(\theta) = gm$$

Then, we divide the left sides and set that equal to the quotient of the right side:

$$\frac{F_N \sin(\theta)}{F_N \cos(\theta)} = \frac{F}{gm}$$

$$\tan(\theta) = \frac{F}{gm}$$

$$F = gm\tan(\theta) = 10(5)\tan(37^o) = 37.5\text{ N}\ .$$

Returning to either of the original equations results in

$$F_N \cos(\theta) = gm \quad\rightarrow\quad F_N = \frac{gm}{\cos(\theta)} = \frac{10(5)}{\cos(37^o)} = 62.2\text{ N}$$

MATHEMATICAL JUSTIFICATION

Suppose that A = B and C = D ≠ 0. It should be O.K. to say that

$$\frac{A}{C} = \frac{B}{C}\ ,$$

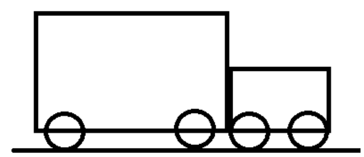
but since C = D,

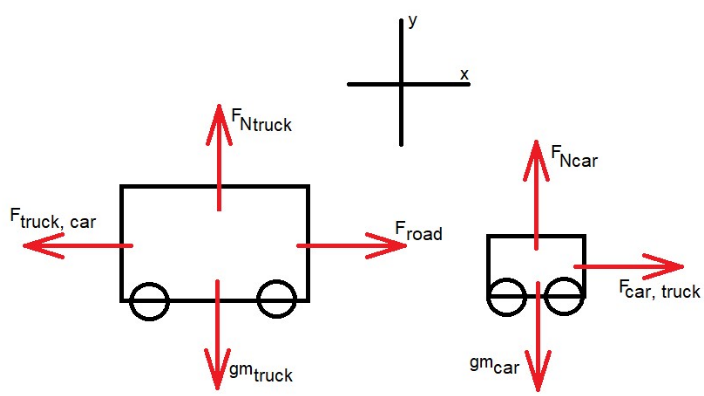
$$\frac{A}{C} = \frac{B}{D}\ .$$

Our next step is to investigate solving problems with more than one body. Remember that only the forces that actually act on an object can affect its motion. Other forces affect the motions of other objects.

EXAMPLE 5-4

Two of my uncles once started one of their cars by pushing it with the other car (It works fairly well with manual transmission cars. Don't try it with automatics.). Suppose Bud's truck has a mass of $m_T = 2500$ kg and Russell's car has a mass of $m_C = 1800$ kg. Bud's truck has a force from the road of 8000 N pushing it forward.[7] If the truck is in contact with the car, what will be the acceleration of the vehicles and what force will each exert on the other?





We have two objects. We should isolate them from each other and from all other objects (such as the earth) so that we can analyze the forces acting on each. Of course, each car has a weight, and a normal force acts on each upward from the ground. For the truck, there is a force forward from the ground and a force backward from the car, while the car experiences a force forward from the truck. Since the presumed acceleration is to the right, we'll use Rule One and choose the coördinate system shown. The third law of motion says that if the truck exerts a force on the car, then the car exerts a force of the same magnitude in the opposite direction, so $F_{TRUCK,\ CAR} = F_{CAR,\ TRUCK}$. We must write a set of second law equations for <u>each</u> object.

Truck	Car
x: $+F_{ROAD} - F_{TRUCK,CAR} = m_{TRUCK}a_x$	x: $+F_{CAR,TRUCK} = m_{CAR}a_x$

[7] We'll discuss the nature of this force later in this section. If you like, for now you could say the truck's tires are pushing the truck forward.

$y\!: +F_{N\ TRUCK} - gm_{TRUCK} = m_{TRUCK}a_y = 0$ $\qquad$ $y\!: \ + F_{N\ CAR} - gm_{CAR} = m_{CAR}a_y = 0$

The y equations are of no use, so we'll add the x equations to eliminate the forces acting between the vehicles.

$$F_{ROAD} - F_{TRUCK,CAR} = m_{TRUCK}a_x$$

$$F_{CAR,TRUCK} = m_{CAR}a_x$$

$$F_{ROAD} = (m_{TRUCK} + m_{CAR})a_x$$

$$a_x = \frac{F_{ROAD}}{m_{TRUCK} + m_{CAR}} = \frac{8000}{2500 + 1800} = 1.86\,\mathrm{m/s^2}\ .$$

Now, we go back to find the contact force. We can use either equation, but why not use the simpler one?

$$F_{CAR,TRUCK} = m_{CAR}a_x = 1800(1.86) = 3348\ \mathrm{N}\ .$$

MATHEMATICAL JUSTIFICATION

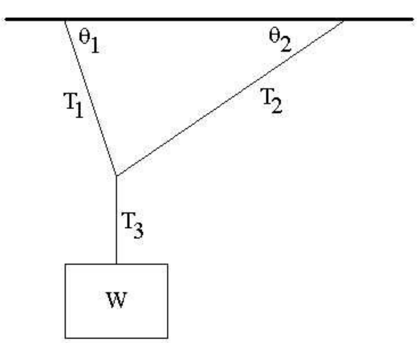
Suppose that A = B and C = D. It should be O.K. to say that

$$A + C = B + C$$

but since C = D,

$$A + C = B + D\ .$$

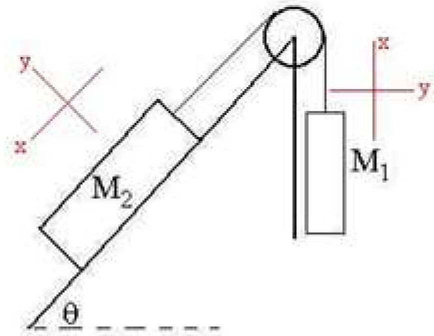
HOMEWORK 5-5



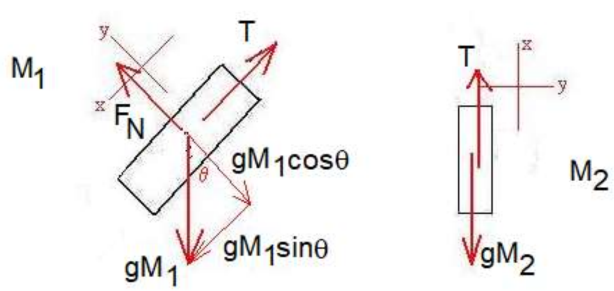
An object with a weight of 250 N is hung from the ceiling as shown. Find the tension in each of the wires if $\theta_1 = 53^{\circ}$ and $\theta_2 = 30^{\circ}$. Hints: There are two objects to investigate, the mass itself, and the knot where the wires meet.

EXAMPLE 5-5

This example introduces a couple more new notions. Consider two blocks as shown with the inclined surface being without friction (whatever that is). Since there are two bodies, we will have to have two free body diagrams and two sets of second law equations. There looks to be a complication in choosing a coördinate system, though; no matter how the x and y axes are oriented, at least one acceleration will need to have two components, and there will have to be several equations relating the accelerations of each block to each other. We can avoid this by using a *fractured coördinate system*. For example, if mass one slides up the incline by one meter , mass two must descend exactly one meter. Remember that our strings are inextensible. By using the system shown below, we can minimize the tedium of relating all the necessary quantities and use simply Δx, v, and a to describe the motions of the masses along their respective x axes, while asserting that there is no motion in the respective y. We will also assume that the wheel has no effect other than to change the direction of the string; it is massless and frictionless. I will use the term *magic* to mean this.[8] The combination of a magic string with a magic wheel means that the tension is the same at both ends of the string. The problem is, find the acceleration of the masses and the tension in the string. When a problem is worded like this, the expectation is that the tension will not appear in the answer for the acceleration, and the acceleration will not appear in the answer for the tension; the answers should be in terms of only those quantities given in the problem, plus some obvious ones such as g.



Let's do free body diagrams. Note that the angle marked θ in this diagram is the same as the original angle of inclination.



So, write a second law equation for each mass:

$$M_1 \;\; x: \; + gM_1 \sin(\theta) - T = M_1 a_x$$

$$M_1 \;\; y: \; + F_N - gM_1 \cos(\theta) = M_1 a_y = 0 \; ,$$

$$M_2 \;\; x: \; - gM_2 + T = M_2 a_x$$

$$M_2 \;\; y: \;\; \text{No Forces} \; .$$

This last ‘equation’ is written to assume me that you have checked the y-direction for M_2 and there is just simply nothing going on there.

Once again, we’re going to add the x equations:

[8] In Section 8, we will deal with non-magic wheels.

$$gM_1 \sin(\theta) - T = M_1 a_x$$

$$-gM_2 + T = M_2 a_x$$

$$gM_1 \sin(\theta) - gM_2 = (M_1 + M_2) a_x \ .$$

This is an efficient way to eliminate the tension terms. The acceleration is then

$$a_x = \frac{M_1 \sin(\theta) - M_2}{M_1 + M_2} g \ .$$

Let's stop and think about whether this makes some sense. The acceleration should be inversely proportional to the total mass (check). If M_1 is much larger then M_2, the blocks should slide to the left, and vice versa. Is there some condition whereby the blocks could just balance? Seems O.K.

Now to find the tension. Substitute the acceleration result back into the simpler of the two x equations:

$$-gM_2 + T = M_2 \left(\frac{M_1 \sin(\theta) - M_2}{M_1 + M_2} g \right) ,$$

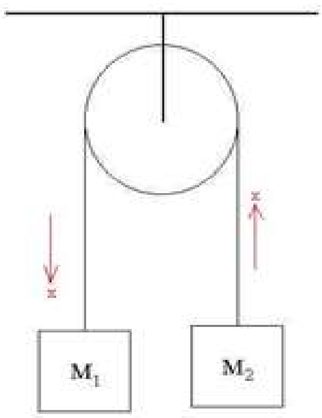
$$T = M_2 \left(\frac{M_1 \sin(\theta) - M_2}{M_1 + M_2} g \right) + gM_2 \ .$$

Technically, this expression meets the requirement I put on the solutions. If this were an exam question, for example, I would accept it. It is possible, though, to make it prettier.

EXERCISE 5-1

Simplify the expression above for the tension. Get in some practice with algebra.

EXERCISE 5-2

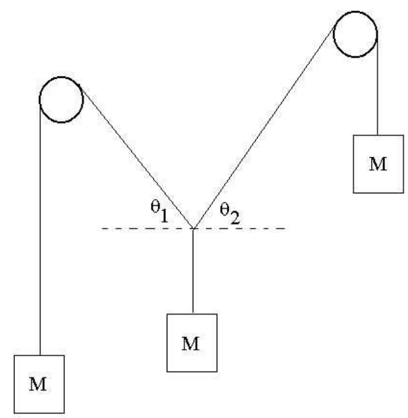


Consider the *Atwood's Machine*, comprising two masses connected by a massless string over a magic wheel. Find the acceleration of the masses and the tension in the string. Let M_1 be 5 kg and M_2 be 7 kg. Give yourself no more than one minute to obtain the correct answer.

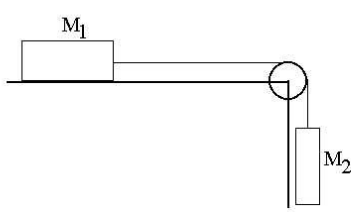
Note the difference between 'answer' and 'solution.' The solution would take much longer than a minute to produce.

HOMEWORK 5-6

Three identical masses are hung from strings as shown in the figure. The strings and wheels are magic. Find the two angles θ_1 and θ_2. HINT: again, consider the knot as a separate object.



HOMEWORK 5-7



A box (M_1) sits on a frictionless table and is connected by a magic string over a magic wheel to another mass (M_2), as shown. Find the acceleration of the masses and the tension in the string.

Let's now consider some objects in uniform circular motion. To review, such an object, moving in a circle at constant speed, has an acceleration towards the center of the circle (centripetal acceleration) with its magnitude given by

$$\mathbf{a_C} = \boldsymbol{\omega}^2\mathbf{r} = \frac{\mathbf{v}^2}{\mathbf{r}} \; .$$

As we have seen earlier in this section, accelerations are caused by forces. There must, therefor, be a force or at least a force component towards the center of the circle. We call this of course a *centripetal force*. This sometimes causes confusion; better to call this 'a force that acts centripetally.' The reason I say this is that, often, students will correctly draw in the force that acts centripetally, but then add in an additional centripetal force as if it is separate from the actual forces. The general rule for this is, if you can't identify what is exerting the force, it's probably not actually there.

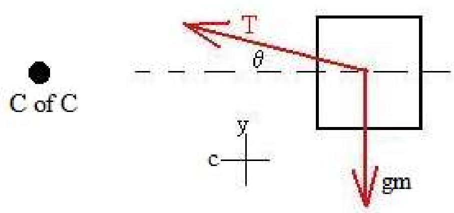
Some forces of course are directed away from the center of the circle; we refer to these as *centrifugal forces*. This is also something to be careful about.

To be consistent with Rule One, we will make one axis in the direction of the acceleration, that is, we will place the c-axis as positive toward the center of the circle. Centripetal forces will have a plus sign inserted in front of their magnitudes, and centrifugal forces a negative sign.

DISCUSSION 5-12

Suppose that I swing a small mass m on a string around in a horizontal circle of radius r. What forces are acting on the mass? Perform the checklist. Is there weight? Is the mass touching anything? Is there a string? Draw a free body diagram for the mass. What if I slow the speed of the object down, does that change your ideas about the directions?

EXAMPLE 5-6



Consider the problem in the discussion above. Find the relationship among theta, the speed, and the radius of the circle. Draw the free body diagram, indicate the center of the circle (C of C) and choose the coordinate system so that the c-direction is in the direction of the acceleration toward the center. We'll then make y be vertical. Then, we'll need to decompose the tension into c and y components. The second law equations are then

$$\text{c:}\quad T\cos\theta = ma_C = m\frac{v^2}{r}$$

$$\text{y:}\quad T\sin\theta - gm = ma_y = 0 \quad .$$

Can you see why the string must be at an angle above the horizontal? If it were not, there would be a net downward force and the mass would accelerate downwards. Let's re-arrange and divide the y-equation by the c-equation:

$$T\sin\theta = gm$$

$$T\cos\theta = m\frac{v^2}{r}$$

$$\tan\theta = \frac{gr}{v^2} \quad .$$

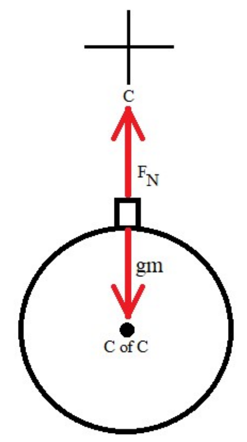
With this result, we see that the angle of the string will decrease as the speed increases. How quickly would the mass need to move to make the string horizontal?

EXAMPLE 5-7

Suppose that you're riding a Ferris wheel of radius 20 m. The operator decides to have fun with the rubes and speeds up the wheel. At how many revolutions per minute would the wheel need to spin in order for you to feel apparently weightless when at the top of the wheel?

First, let's recall what apparently weightless means. It does not mean there is no weight; we can't just shut that off. Like in the discussion of the elevator above, it means that the normal force of contact between you and your seat is zero and you are in free-fall. Second, since revolutions per minutes is angular speed, we'll use that form for the centripetal acceleration. The weight is pointing toward the center of the circle but the normal force is away from the center, so

$$+gm - F_N = ma_C = m\omega^2 r \ .$$

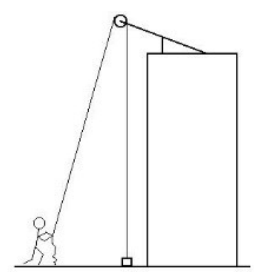


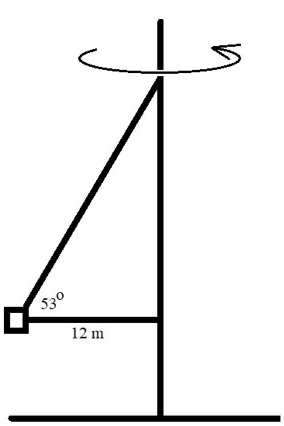
Setting F_N to zero and solving for omega,

$$\omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{10}{20}} = 0.71\frac{\text{rad}}{\text{s}} \times \left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right) \times \left(\frac{60\ \text{s}}{1\ \text{min}}\right) = 6.75\ \text{rev/min}.$$

HOMEWORK 5-8

Suppose you want to lift a 12 kg mass from the ground to an altitude of 15 meters as quickly as possible. The problem is that the rope will break if its tension exceeds 160 N. What's the shortest amount of time in which the mass can be lifted without the rope breaking?





HOMEWORK 5-9
A ride at the firemen's field days (carnivals, down here) comprises a seat connected to a central column by a horizontal strut and a strut connected at a 53° angle, as shown. The lower strut is 12 meters long. If the seat plus passenger has a mass of 120 kg and the ride rotates at 2.4 revolutions per second, what is the tension in each strut?

Pseudo-forces*

As the term suggests, these are forces that don't actually exist. Suppose you're sitting in your car at a red light. When the light turns green, you accelerate forward. You may feel as if there is a force pushing you back into your seat. If you have some trinket hanging from your rearview mirror, you may think that there is something pulling it backwards. These forces don't actually exist. What's actually happening is that the car accelerates forward and exerts a force forward on you to move you forward along with itself. You sink into the seat, the sensors in your body feel your layers being squished together, and your body interprets this as a force pushing backwards. VIDEO.

Similarly, when a car rounds a corner to the right, as an example, you may feel as if you are being pushed outward against the door. In reality, the door is curving to the right while your body has a propensity to move in a straight line. The door is actually pushing you to the right, helping you to move in the circle along with the car.

As you may guess, pseudo-forces are imagined by observers who are in an accelerating frame of reference. Generally, if you can't identify the source of a force, it's probably actually a pseudo-force.

HOMEWORK 5-10

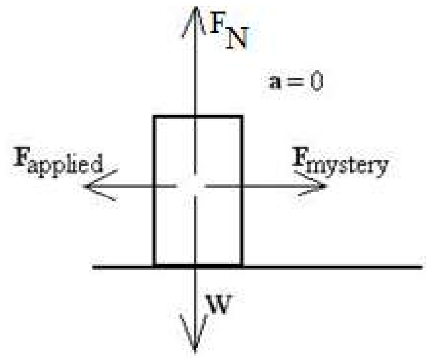
An ornament (m = 0.3 kg) hangs from the rearview mirror of a car. When the car accelerates forward along a horizontal road, the die appears to swing backward so that the string supporting it makes an angle of 6° with the vertical. What is the acceleration of the car? HINT: We're not concerned about *how* the die swung back, only that it *has* swung back.

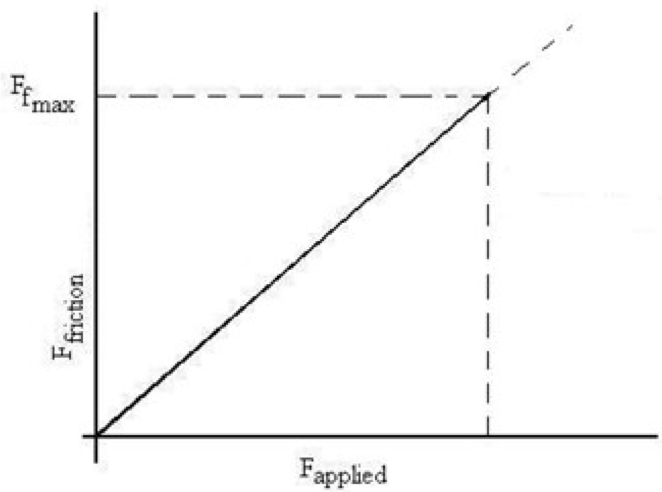
DISCUSSION 5-13

Can you think of a way this effect could be used as, for example, a safety device in an automobile?

Friction

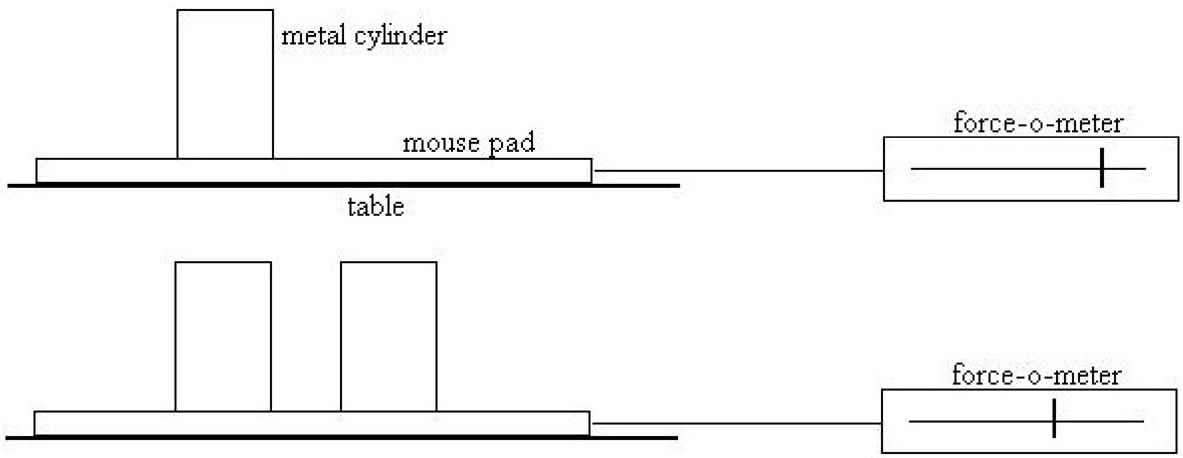
Now, we consider the fourth force, another contact force, that we've been dancing around since Section 2. *Friction* occurs at the interface between two surfaces and is directed along the surface (not perpendicular to it, as for normal contact forces), opposite to the direction in which the surfaces are sliding or want to slide. There are two types of friction that we will consider: *kinetic* and *static*. It is a common misconception that an object must be stationary to experience static friction or moving to experience kinetic friction. What is important is whether the surfaces in question are sliding against one another or not. Let's start by considering an object at rest on the desk; clearly the sum of the forces acting on this object (weight and normal force from the desk) is zero, since there is no acceleration. If we apply a small force horizontally to the object, we may be mildly surprised that it does not accelerate; if the second law is to remain correct, there must be yet another mystery force acting oppositely to our applied force that causes the total horizontal force to be zero (second law, $F_{appl} - F_{mystery} = ma = 0$). What's more, the magnitude of that force <u>changes</u> as we change our applied force; it's always just big enough to cancel our force. That is, if we apply 2 newtons, it applies 2 newtons, if we apply 5 newtons, it applies 5 newtons. We'll call this force *friction*. A graph of this situation might look like this, a line of slope one passing through the origin:





Furthermore, we see that, if we continue to increase our applied force, there comes a point at which this frictional force reaches a maximum value; we know this because we can apply enough force to make the object move, and that requires a <u>net</u> non-zero force. How big is this maximum frictional force and what quantities determine its value?

CHEESEY EXPERIMENT 5-5



In this experiment, we placed a one kg metal cylinder atop a mouse pad 'sled' and slowly increased the applied force as measured by our force-o-meter. If we apply a non-zero force and the mass doesn't move, then from the second law, we know how much frictional force is applied. The point of interest here, of course, is the value of the applied force for which the sled just begins to move, which is then also the maximum frictional force. In this case, that force was 2 N. What if the mass is doubled to 2 kg? What do you think the maximum force will be? It was in fact 4 N.

DISCUSSION 5-14

On what parameters of this experiment do you think the maximum friction force depends? Suppose I place my hand on the mousepad and repeat the experiment to obtain 1.6 N. I repeat the experiment but this time obtain 26 N. Did the mass of my hand change? What did change? Is this notion consistent with what we saw in the first part of the experiment? When I doubled the mass, what else doubled?

It appears that the magnitude of the maximum possible frictional force is proportional to the magnitude of the normal force pushing the two surfaces together. We usually use the Greek letter μ (mu) as the constant of proportionality and call it the *coëfficient of friction*:

$$F_{f\,MAX} = \mu F_N \ .$$

EXPERIMENT 5-5 CONTINUED

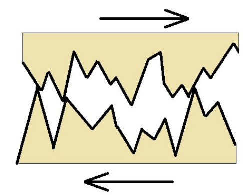
Let's flip the mousepad over and repeat. The results are listed in the table.

Normal Force	Maximum Frictional Force (pad right side up)	Maximum Frictional Force (pad upside down)
0 N	0 N	0 N
9.8 N	2N	3 N
19.6 N	4 N	6 N

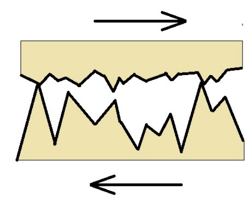
coëfficient of friction	0.20	0.31

You may notice that the coefficient is different for the two parts of the experiment. What is different between the two situations?

The value of the coëfficient of friction is always for a pair of surfaces. We can't say that the coefficient for the tabletop is 0.4, it must be the value for the tabletop and the bottom of the mousepad. Since the surface of the mousepad is rougher than the top, that coëfficient will be higher.



What is the nature of friction? In this case, it is once again the springiness of the bonds between atoms. Surfaces are never perfectly smooth, and comprise 'valleys' and 'hills.' As the surfaces try to slide against one another, the points of lateral contact will exert forces to prohibit the sliding. If one of the surfaces is smoother, there will be fewer points of contact, and it will require less force to initiate the slide. You can try this yourself by using your fingers.



DISCUSSION 5-15

Does friction disappear once the surfaces start to slide? Suppose that you are moving a couch across a floor. Do you need to continue to apply a force in order to keep the couch moving once it's started? Is it harder to get the couch to start to move, or to keep it moving?

So, it appears that there are two types of friction. One when the surfaces are not sliding, as discussed above, and one when the surfaces are already sliding. To distinguish these, let's call the first (above) *static friction* and the new one *kinetic friction*. Let's repeat the experiment above for sliding surfaces.

EXPERIMENT 5-5 CONTINUED

This time, we'll get the mass moving, then apply enough force to keep it moving at a constant velocity. Then the acceleration will be zero and once again, the applied force will be the same size as the frictional force.

Normal Force	Frictional Force (pad right side up)	Frictional Force (pad upside down)
0 N	0 N	0 N
9.8 N	2.5 N	3 N
19.6 N	5.0 N	10 N
coëfficient of friction	0.26	0.51

We see that the frictional force is proportional to the normal force. However, the proportionality constants are different from their respective values in the static case. We'll need to distinguish them as μ_S and μ_K:

$$F_{fK} = \mu_K F_N \ .$$

We will assume that, unlike static friction, kinetic friction always has this value. Remember that in the first half of the experiment, we found the maximum value of the static friction. At this point, it's worth doing a brief review.

- There is a force of contact called friction which acts along the interface of two objects (as opposed to perpendicular to the interface, as for the normal force of contact).

- If the surfaces are not sliding against one another, we call the friction static. This static friction force is only as big as it needs to be to prevent the surfaces from sliding against one another, but only up to a maximum value that depends on the natures of the two surfaces and on how hard they are being pushed together:

$$F_{fS} \leq \mu_S F_N \ .$$

  Because of the inequality in the relationship for static, we concentrate on situations where the surfaces are 'about to slide,' or there is some similar condition so that we know that we are at the critical point when the equality holds true. You should justify this when to do homework or exam problems.

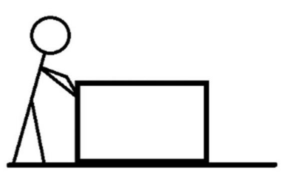
- If the surfaces are already sliding, we have kinetic friction, in which case

$$F_{fK} = \mu_K F_N \ .$$

DISCUSSION 5-16

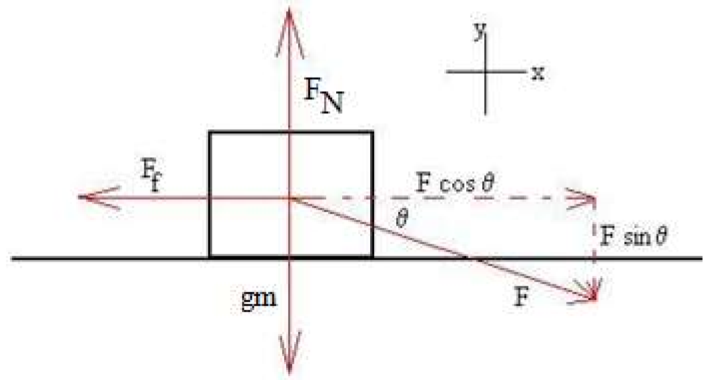
What are the units for the coefficient of friction?

EXAMPLE 5-8



You're pushing a 800 N box at constant velocity across a floor by applying a 200 N force F applied at an angle of 25° below the horizontal. What is the coëfficient of kinetic friction between the box and the floor?

Because of the wording of the problem (F keeps the box moving), we assume that the block is moving toward the right. Correspondingly, the frictional force is to the left. The velocity is constant, so all components of the acceleration are zero. We therefor make use of Rule Two in choosing a coördinate system. The second law equations are then



$$+F\cos\theta - F_{FK} = ma_x = 0$$

$$-F\sin\theta + F_N - gm = ma_y = 0\ .$$

We also need an equation for the friction. Since this is kinetic friction, there is no question that we use the equal sign:

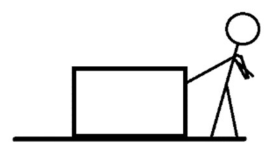
$$F_{fK} = \mu_K F_N\ \ .$$

The solution is fairly straightforward substitution:

$$\mu_K = \frac{F_{fK}}{F_N} = \frac{F\cos\theta}{gm + F\sin\theta} = \frac{200\cos(25^0)}{800 + 200\sin(25^o)} = 0.20$$

DISCUSSION 5-17

Your boss sees you moving the crate in this manner and says, "You need to work smarter, not harder. Get a rope and pull the crate at 25° above the horizontal." Will this require less force? What physical reasons (in words) would back this up?



EXERCISE 5-3

Go ahead and calculate how much force would be necessary at 25° above the horizontal to keep the crate moving at a constant velocity. Was your boss correct?

DISCUSSION 5-18

What question should you ask yourself next?

EXAMPLE 5-9

Can we find what angle would result in the lowest required force to keep the crate moving at a constant velocity? The force necessary to keep the crate moving as a function of the applied angle (see Exercise 5-3) is given by

$$F = \frac{\mu_K gm}{\cos\theta + \mu_K \sin\theta} = \mu_K gm(\cos\theta + \mu_K \sin\theta)^{-1}.$$

Set

$$\frac{dF}{d\theta} = -\mu_K gm(\cos\theta + \mu_K \sin\theta)^{-2}(-\sin\theta + \mu_K \cos\theta) = 0 \; .$$

Neither of the first two parts can equal zero, so we're left with

$$\sin\theta = \mu_K \cos\theta \;\; \rightarrow \;\; \mu_K = \tan\theta \;\; \rightarrow \;\; \theta = \arctan(\mu_K) = \arctan(0.2) = 11.3^o \; .$$

Static friction problems are generally harder than kinetic problems. First of course, we may not be at the critical point of the surfaces being just about to slide, and often we don't even know which way they will try to slide. Let's look at a fairly standard problem and see what kinds of questions can be asked and how we might deal with them.

HOMEWORK 5-11

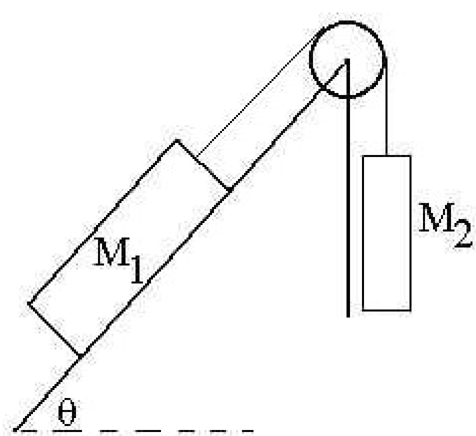
A mover finds that a 120 kg dresser requires a 70 N horizontal force to set it in motion across the floor, but only 55 N to keep it moving with constant velocity. Find the static and kinetic coëfficients of friction between the bottom of the dresser and the floor.

HOMEWORK 5-12

What minimum force is necessary to drag a crate of mass 60 kg across a floor at constant velocity with a rope inclined at 37° above the horizontal? The coëfficient of kinetic friction is 0.7.

EXAMPLE 5-10

Consider once again two blocks connected by a light string over a magic wheel, with one block on a rough incline.



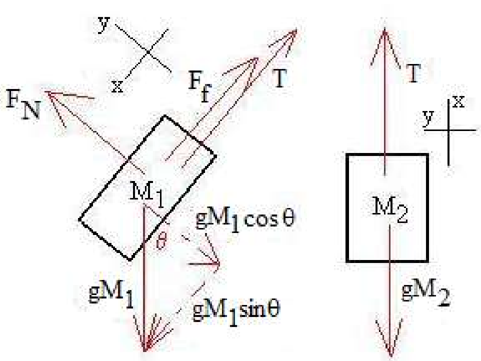
What kinds of questions could be asked? One might wonder

- what is the largest value of M_1 (or the smallest value of M_2) for which M_1 will not slide down the plane?
- what is the smallest value of M_1 (or the largest value of M_2) for which M_1 will not slide up the plane?

- what is the smallest value of μ_S for which M_1 will not slide down (or up) the plane?
- what is the smallest (or largest) value for θ for which M_1 will not slide up (or down) the plane?
- what is the direction and magnitude of the acceleration, assuming that the blocks are in motion?
- about many things not listed here.

Many of these types of problems require us to assume or guess which way the blocks would slide if there were no friction, since we need to be able to assign a direction to the frictional force that would keep that from happening. Sometimes, the direction is obvious, other times we will need to solve the problem first with $\mu_S = 0$. Let's take a run at several of these.

Write the second law, assuming that the blocks are moving, or about to move, with M_1 sliding down the incline. Keep in mind, we may well be wrong about this. Even worse, the blocks may be moving one way but accelerating the other!



For M_2, we have

$$+T - gM_2 = M_2 a_x$$

No Forces

For M_1,

$$-T - F_f + gM_1 \sin\theta = M_1 a_x$$

$$+F_N - gM_1 \cos\theta = M_1 a_y = 0$$

$$F_f = \mu F_N \ .$$

Note that I haven't specified which type of friction, although if it's static, the system must be just about to move. Nor have I specified zero acceleration. I have assumed that the frictional force is directed up the incline. Let's do some math.

$$F_f = \mu F_N = \mu(gM_1 \cos\theta)$$

Substitute this into the M_1 x equation.

$$-T - \mu\, gM_1 \cos\theta + gM_1 \sin\theta = M_1 a_x$$

Then, as usual, add the M_2 x equation to eliminate the tension.

$$-T - \mu\, gM_1 \cos\theta + gM_1 \sin\theta = M_1 a_x$$

$$+T - gM_2 = M_2 a_x$$

$$-gM_2 - \mu\, gM_1 \cos\theta + gM_1 \sin\theta = (M_1 + M_2)a_x$$

$$g\big(-M_2 + M_1(\sin\theta - \mu\, \cos\theta)\big) = (M_1 + M_2)a_x$$

Let's answer some of the questions that were listed above.

What is the largest value of M_1 for which M_1 will not slide down the plane? Since the masses are not yet moving, $a_x = 0$, the friction is static and at the critical point, and the equation simplifies to

$$M_2 - M_1(\sin\theta - \mu\, \cos\theta) = 0$$

$$M_{1MAX} = \frac{M_2}{\sin\theta - \mu\, \cos\theta}$$

What is the smallest value of M_1 for which M_1 will not slide up the plane? We don't have to redo the entire problem. The only thing that changes from the first question is the direction of the frictional force, and we can fix that with a mathematical trick by replacing μ with $-\mu$. So, we immediately know that

$$M_{1MIN} = \frac{M_2}{\sin\theta + \mu\, \cos\theta}\ .$$

What is the smallest value of μ_S for which M_1 will not slide down (or up) the plane? Once again, the acceleration is zero and the friction is static, so

$$M_2 = M_1(\sin\theta - \mu\, \cos\theta)$$

$$\mu_{S\,MIN} = \frac{\sin\theta - \frac{M_2}{M_1}}{\cos\theta}\ .$$

What is the smallest value of μ_S for which M_1 will not slide up the plane? We'll use our math trick again, since the only difference in the equations will be the direction of the frictional force.

$$\mu_{S\,MIN} = -\left(\frac{\sin\theta - \frac{M_2}{M_1}}{\cos\theta}\right) = \frac{\frac{M_2}{M_1} - \sin\theta}{\cos\theta}\ .$$

I'm sure you're getting the idea here. Now, what if the masses start from rest and then slide? What would be the acceleration? That's going to depend on which way they slide. So, first we need to find the direction of acceleration without friction, then put it back in to find the actual acceleration:

$$a_{x\text{ NO FRICTION}} = \frac{-M_2 + M_1 \sin\theta}{(M_1 + M_2)} g$$

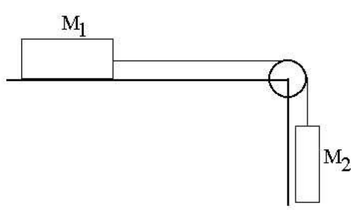
$$a_x = \frac{\left(-M_2 + M_1(\sin\theta - \mu_K \cos\theta)\right)}{(M_1 + M_2)} g$$

If the sign of this ‘no friction’ acceleration is positive, we’re already O.K. and we return to the solution as given. If the ‘no friction’ acceleration is negative, then we need to reverse the sign of the coefficient of friction, then compute:

$$a_x = \frac{\left(-M_2 + M_1(\sin\theta + \mu_K \cos\theta)\right)}{(M_1 + M_2)} g$$

Of course, if you’re told which direction they are moving in, you just pick the correct sign for μ_K.

HOMEWORK 5-13

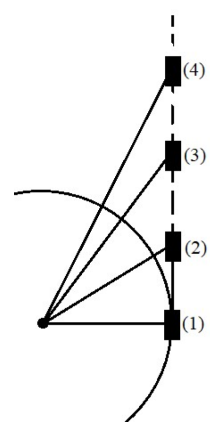


A 15 kg mass (M_1) is connected by a magic string over a magic wheel to a 5 kg mass (M_2) as shown in the figure. When the masses are released, M_2 falls from rest a distance of 2.2 meters in 3 seconds. What is the coëfficient of kinetic friction between M_1 and the table surface?

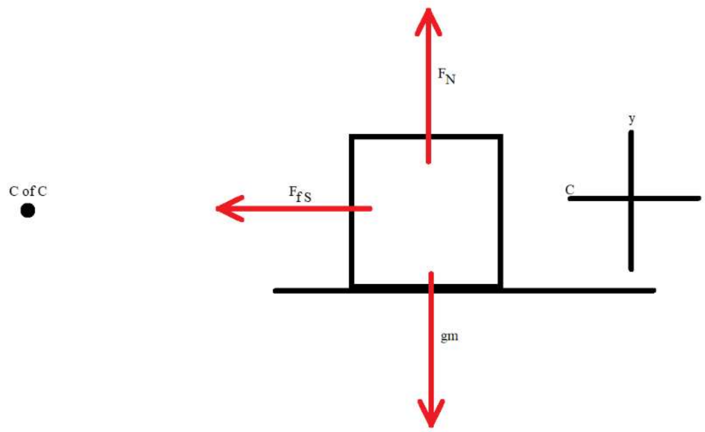
EXAMPLE 5-11

Suppose you’re bored early on a Sunday morning and you decide to drive your 1800 kg car in circles in the parking lot at the local mall. How quickly can you drive the car in a circle of radius 50 m?

So, what forces act on the car? Obviously, there is weight. There is a normal force from the pavement upward, and there is friction. If you’ve tried this on an icy surface, you know the car will simply travel in a straight path. In what direction is the friction? Well, the car is ‘trying’ to move in a straight path, which means it’s trying to slide away from the center of the circle. Since the friction opposes that attempt, it must point toward the center of the circle. You can even feel this force with your hands VIDEO. Which type of friction is this, static or kinetic? Be careful! Let’s say the coëfficient of friction is 0.85.



Let’s do a free body diagram.



Rule One tells us to place the c-axis toward the center of the circle. The second law equations are

$$c:\ +F_{fS} = ma_C = m\frac{v^2}{r}$$

$$y:\ +F_N - gm = ma_y = 0$$

$$F_{FS} = \mu_S F_N \quad \text{crit. sit.}$$

We can use the equality in the static friction equation because we're looking for the maximum speed, *i.e.*, the tires are just about to slip. Re-arranging and substituting,

$$v^2 = \frac{r\,F_{fS}}{m} = \frac{r\,\mu_S F_N}{m} = \frac{r\,\mu_S gm}{m} = r\,\mu_S g = 50(0.85)10 = 425$$

$$v_{MAX} = \sqrt{425} = 20.6\ \text{m/s}\ .$$

Notice that the mas of your car didn't matter.

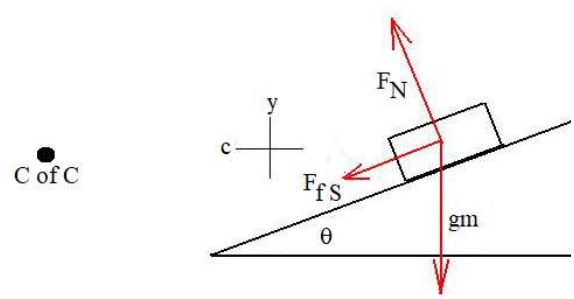
DISCUSSION 5-19

What does MDOT do to help you get around sharp curves on the highway? Think especially about cloverleafs. How does this help?

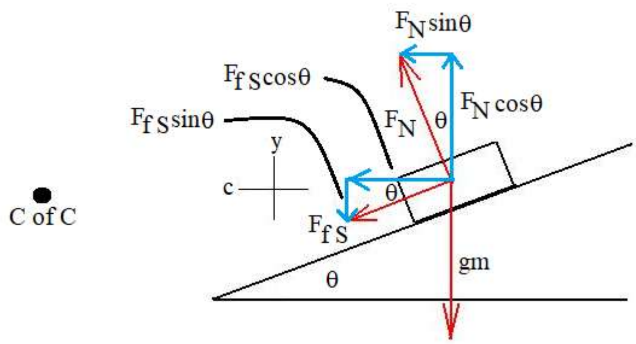
EXAMPLE 5-12

Turn 1 at the Talladega Superspeedway is inclined at a 33° angle from the horizontal and has a radius of 330 m (this depends of course on which lane you are in). Brand new tires have a coëfficient of static friction with the track surface of 1.3. What is the theoretical maximum speed a 1,636 kg car can travel and still negotiate this turn?

Of course, we begin with a free body diagram. As in the previous example, this car would tend to move toward the outside of the circle, and so the static frictional force would act to oppose this sliding and so push it toward the center. Following Rule One, the c-axis is toward the center of the circle and then the y-axis is vertical. We will need to decompose both the frictional force and the normal force.



The second law equations become:



$$c:\ +\ F_{fS}\cos\theta +\ F_N\sin\theta = ma_C =\ m\frac{v^2}{r}$$

$$y:\ + F_N\cos\theta -\ F_{FS}\sin\theta - gm = ma_y = 0$$

$$F_{fS} =\ \mu_S F_N \quad \text{crit. sit.}$$

Let's re-arrange, substitute, and divide.

$$+\ \mu_S F_N\cos\theta +\ F_N\sin\theta = m\frac{v^2}{r}$$

$$+F_N\cos\theta -\ \mu_S F_N\sin\theta = gm$$

$$\frac{\mu_S F_N\cos\theta +\ F_N\sin\theta}{+F_N\cos\theta -\ \mu_S F_N\sin\theta} = \frac{m\frac{v^2}{r}}{gm}$$

$$\frac{\mu_S\cos\theta +\ \sin\theta}{\cos\theta -\ \mu_S\sin\theta} = \frac{v^2}{gr}$$

At this point, of course, we could solve for any of the variables contained in this relationship. Proceeding to find the maximum speed results in

$$v_{MAX} = \sqrt{gr\frac{\mu_S\cos\theta +\ \sin\theta}{\cos\theta -\ \mu_S\sin\theta}} = \sqrt{10(330)\frac{1.3\cos 33^o +\ \sin 33^o}{\cos 33^o -\ 1.3\sin 33^o}} = 203\ \mathrm{m/s}\,.$$

The record is about 96 m/s.

Suppose in the previous example, we had wanted to find the slowest possible speed so as not to slide to the center of the track. Since the only thing different about the problem is the direction of the frictional force, we can take this result and flip the sign of the coefficient of friction:

$$v_{MIN} = \sqrt{gr\frac{-\mu_S\cos\theta +\ \sin\theta}{\cos\theta +\ \mu_S\sin\theta}}\ .$$

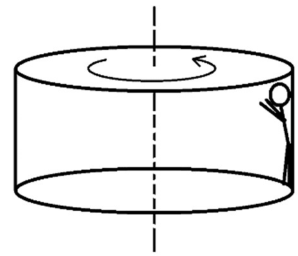
With these particular numbers, I expect that we'll be taking the square root of a negative number, which means the car could be parked on the incline and not slip, but with a coëfficient corresponding to an icy surface, there may well be a real minimum speed.

HOMEWORK 5-14

A dime (m = 2 grams) sits at the edge of the platter of a record player (They've made a comeback, so I know you know what that is). What is the minimum coëfficient of static friction that will keep the coin on the platter as it spins A standard LP is 30 cm in diameter and rotates at $33^1/_3$ revolutions *per* minute.

HOMEWORK 5-15

Again at the field days, a patron enters a circular room of radius 5 m. The room starts to spin and speeds up to 7 radians/second, at which time the floor drops away. What must be the minimum coëfficient of static friction so that the passenger does not slide down to a certain death?



Outside the Safe Zone*

Let's take a crack at a problem that does have drag. Consider a small metal ball of mass m dropped from a great height. In the absence of air, it will accelerate downward uniformly at some value around 9.8 m/s^2. The drag force is commonly assumed to be proportional to the speed of an object, and of course depends on the size, shape, and orientation of the object as well as the properties of the fluid though which it is travelling. The direction is opposite to the motion of the object through the fluid. Let's write NII for the vertical direction, with down positive:

$$+\mathrm{gm} - \mathrm{bv} = \mathrm{ma}.$$

Qualitatively, we can see that, just after release when the speed is extremely small, the acceleration will be equal to a_g. However, as the ball falls and picks up speed, the acceleration will become less and less (still downward though of course). Eventually, the ball acquires speed v = gm/b, at which point the acceleration becomes zero, and the velocity accordingly becomes constant. This ultimate speed is known as the *terminal velocity* of the ball. Of course, this value will vary from object to object and even on the orientation of the object, if it is not spherical.

Let's be a bit more analytical about this problem and solve for the velocity as a function of time. In doing so, we'll find the solution to a problem that re-occurs often in this course.

Re-arranging the NII equation above:

$$\mathrm{a} = \frac{d\mathrm{v}}{d\mathrm{t}} = \mathrm{g} - \frac{\mathrm{b}}{\mathrm{m}}\mathrm{v}$$

$$\frac{d\mathrm{v}}{\mathrm{g} - \frac{\mathrm{b}}{\mathrm{m}}\mathrm{v}} = d\mathrm{t}$$

$$\int_0^{v} \frac{dv}{g-\frac{b}{m}v} = \int_0^{t} dt \quad .$$

I love u-substitutions:

$$u = g - \frac{b}{m}v \qquad du = -\frac{b}{m}dv \quad \rightarrow \quad dv = -\frac{m}{b}du \quad .$$

$$-\frac{m}{b}\int_{g}^{g-\frac{b}{m}v} u^{-1}\, du = \int_0^{t} dt$$

$$-\frac{m}{b}\left(\ln\left(g-\frac{b}{m}v\right) - \ln(g)\right) = t \quad .$$

$$\ln\left(1-\frac{b}{gm}v\right) = \frac{-bt}{m} \ .$$

$$1-\frac{b}{gm}v = e^{\frac{-bt}{m}} \quad .$$

$$v(t) = \frac{gm}{b}\left(1-e^{-\frac{bt}{m}}\right) .$$

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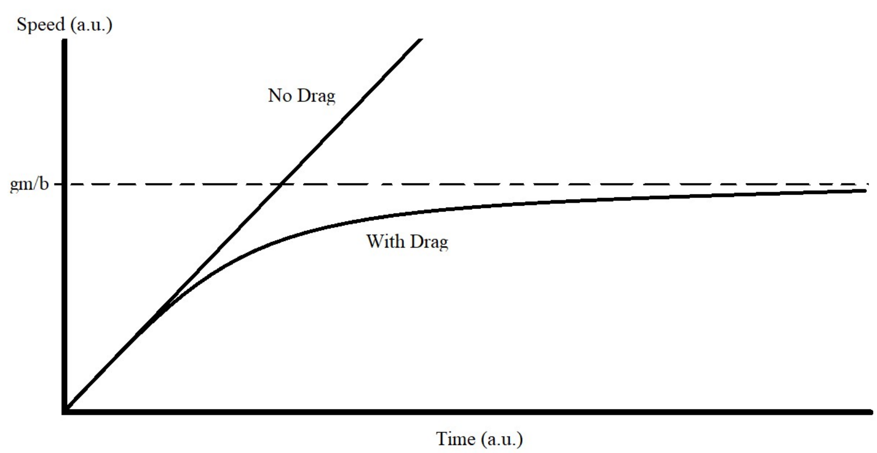


EXERCISE 5-1 Solution

$$T = M_2\left(\frac{M_1\sin(\theta) - M_2}{M_1 + M_2}g\right) + gM_2$$

$$T = \left(\frac{M_1M_2\sin(\theta) - M_2^2 + M_2(M_1 + M_2)}{M_1 + M_2}\right)g$$

$$T = \left(\frac{M_1M_2\sin(\theta) - M_2^2 + M_2M_1 + M_2^2}{M_1 + M_2}\right)g$$

$$T = \left(\frac{M_1M_2\sin(\theta) + M_2M_1}{M_1 + M_2}\right)g$$

$$T = \left(\frac{M_1M_2}{M_1 + M_2}\right)(\sin(\theta) + 1)g$$

EXERCISE 5-2

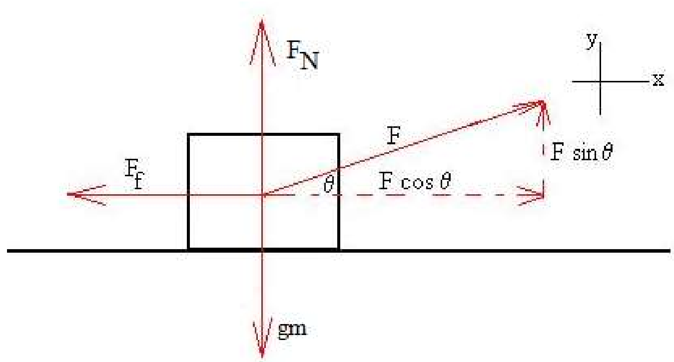
Well, you just did this problem. In this case, though, the angle of the 'incline' is 90°.

$$a_x = \frac{M_1\sin(\theta) - M_2}{M_1 + M_2}g = \frac{5\sin(90^o) - 7}{5 + 7}10 = -1.67\ m/s^2.$$

$$T = \left(\frac{M_1M_2}{M_1 + M_2}\right)(\sin(\theta) + 1)g = \left(\frac{5(7)}{5 + 7}\right)(\sin(90^o) + 1)10 = 58.33\ N$$

EXERCISE 5-3 Solution

The second law equations are almost the same; only the vertical component of the applied force changes direction (it will presumably also change magnitude).



$$+F\cos\theta - F_{FK} = ma_x = 0$$

$$+F\sin\theta + F_N - gm = ma_y = 0\ .$$

$$F_{fK} = \mu_K F_N\ .$$

This solution is a bit more tedious.

$$F\cos\theta = F_{FK} = \mu_K F_N = \mu_K(gm - F\sin\theta) = \mu_K gm - \mu_K F\sin\theta$$

$$F\cos\theta + \mu_K F\sin\theta = \mu_K gm$$

$$F\,(\cos\theta + \mu_K \sin\theta\,) = \mu_K g m$$

$$F = \frac{\mu_K g m}{\cos\theta + \mu_K \sin\theta} = \frac{(0.2)800}{\cos(25^o) + \mu_K \sin(25^o)} = 161.5\ \text{N}$$