SECTION 7 – THE THIRD PICTURE

We've looked at the motions of objects using two outwardly different, but ultimately identical, points of view: forces and accelerations, and work and energy. We know that they are the same, since we derived the work-energy theorem using Newton's Second Law and a couple of definitions. Now, we'll introduce yet another picture which we may, or may not, find convenient to use on certain classes of problems.

CHEESY EXPERIMENT 7-1

Consider a small toy car sitting on a table at a spot marked ‘X’; we’ll assume the wheels make its contact with the table frictionless. Observes the car closely. Now, observe the car as it travels through point X. Is it fair to say that the car possesses some quality or property when it’s moving through X that it lacks when stationary? How did the car acquire that property?

DISCUSSION 7-1

Impulse.mp4

You may notice that the car had the property only after a force acted on it. Indeed, I can also remove the property by applying a force opposite to the motion of the car. This is sounding awfully familiar. In fact, we will approach this in a manner very much like the one we used for work and energy. After the experiment, we agreed to the following:

- There is some quality or property the object possesses when it’s moving through X that it lacks when it’s stationary at X. Of course, we know that the object possesses kinetic energy when it’s moving, but we are measuring the transfer of this new property differently, so we must be transferring something else as well. For want of a better name, let’s call this new property *momentum* (symbol p)
- Momentum is transferred into the object by applying a force. However, the force must act for some period of time. That means that the property is not energy, although clearly energy was also being transferred.
- Transferring momentum into (or out of) an object is a process; let us call the <u>transfer</u> of momentum the *impulse* (J) done on the object. Impulse is <u>not</u> momentum; it is the transfer of momentum.
- The bigger the force, the more momentum is transferred: as F↑, J↑. We might even speculate that J is proportional to F. That would certainly be the simplest relationship consistent with our observations. We could be wrong, of course; perhaps $J \sim F^2$ or F^3. We’ll make the simplest assumption and see if there is a contradiction somewhere in our subsequent experiments.
- The greater the time interval over which the force acted, the more impulse is provided: that is, as Δt ↑,J↑ .We might speculate that J is proportional to Δt.

We may perhaps further speculate that, in the simplest possible scenario, $J = F\,\Delta t. = \Delta p$.

DISCUSSION 7-2

Let's look at the center term of the hypothesis formula above. Which kind of a quantity is force? Then, what about impulse and momentum? Is the bank account analogy appropriate here? Can you think of one that may be more appropriate?

DISCUSSION 7-3

What if the force applied to the object were not constant in time? How might we define the corresponding impulse?

When we considered a similar situation in work-energy, we decided to break the displacement up into small intervals, find the work done over each displacement, and then add those together. Let; try the same thing here. Break the tine interval up into small units *d*t and integrate:

$$\vec{J} = \int \vec{F}(t)\, dt \quad .$$

As in Section 6, we know that there may be more than one force acting on an object at once, each with its own effect on the momentum.

DERIVATION 7-1

$$\vec{J}_{TOTAL} = \sum_n \vec{J}_n = \sum_n \int \vec{F}_n(t)\, dt \; = \int \sum \vec{F}_n(t)\, dt = \int \left(\sum \vec{F}_n(t)\right) dt = \int m\,\vec{a}\, dt$$
$$= m \int \frac{d\vec{v}}{dt} dt = m \int d\vec{v} \; = m\,\Delta\vec{v} = \Delta(m\vec{v}) \; .$$

Keeping in mind that we required that $\vec{J}_{TOTAL} = \Delta\vec{p}$, we may perhaps jump to the conclusion that $\vec{p} = m\vec{v}$. [1]

Now, because we wrote this derivation in terms of vectors, and because we know that if two vectors are equal, then their x, y, and z components must independently be equal, we can treat these problems with momentum as three separate problems, one with the x-components, one with the y-components, and one with the z-components.

DISCUSSION 7-4

It is useful to remember that these quantities, such as momentum or energy, are ones that we have defined. Is momentum a real thing? What about energy?

HOMEWORK 7-1

---

[1] As always, we could be wrong and perhaps $\vec{p} = m\vec{v} + \vec{A}$, where $\vec{A}$ is some constant that subtracts out when we find the change in $\vec{p}$. For now, let's assume the simplest case that $\vec{A} = 0$.

What is the ratio of the magnitude of the momentum of a 3kg mass moving at 3 m/s to that of a 2 kg mass moving at 4 m/s? What is the ratio of the respective kinetic energies? NOTE: You should find that one of the objects has more momentum, while the other has more kinetic energy. How is that possible?

HOMEWORK 7-2

Show that the kinetic energy of an object can be written in terms of the magnitude of the momentum as $K = p^2/2m$.

EXAMPLE 7-1

A constant force of 14 N acts on an initially stationary object (mass 6 kg) for 8 seconds. If this was the only force acting on the object, what is the impulse? What is the final speed of the object?

Call the direction of the force the +x direction.

$$\vec{J}_{TOTAL} = \vec{F}\,\Delta t = 14(8) = \text{112 Ns in the + x direction} \quad .$$

Note that, unlike for work/energy, there is no special unit for impulse/momentum. Typically, impulse is newtons seconds, while momentum is kilogram meters/second.

Then,

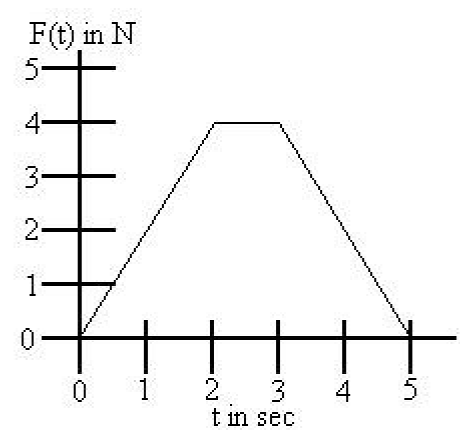
$$\vec{J}_{TOTAL} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i \ ,$$

$$\vec{v}_f = \vec{v}_i + \frac{\vec{J}_{TOTAL}}{m} = 0 + \frac{112}{6} = 18.7 \text{ m/s in the + x direction} \ .$$

HOMEWORK 7-3

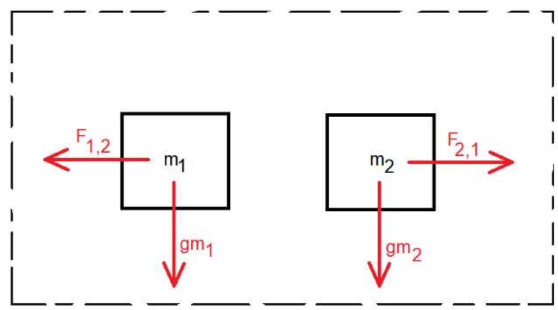
Suppose F(t) shown in the figure is the net force acting in the +x direction on a particle of mass 2kg. Find

a) the impulse imparted to the object by the force.
b) the final velocity of the object if it had been originally at rest.
c) the final velocity of the object if its initial x velocity had been -2 m/s.



As mentioned several times, each of our 'pictures' is especially well suited to solving a particular type of problem. Newton's second law and the kinematic equations were useful when the forces were constant. The work energy theorem was useful when there were no non-conservative forces doing work (mechanical energy was conserved), and had the advantage of not requiring us necessarily to know the path taken by the object of interest or the time that the trip required. In some special cases, the momentum picture is very useful for examining *collisions*. A collision is when two or more objects interact with one another. They do not need actually to touch one another, as we may think about, say, automobile collisions. They may exert other forces on each other, whether gravitational, electric, nuclear, *et c*.

Before we start, let's define a *system*. You may be familiar with this term from chemistry. A system is just the collection of objects in which we are interested. A *closed system* is one for which the only forces acting on the objects are due to other objects in the system; these are called *internal forces*. An *open system* is one for which some force or forces are due to agencies not included in the system; these are of course *external forces*. We can mentally draw an imaginary box around the system; any force that crosses the box's boundary will be an external force. In the figure, the system comprises mass 1 and mass 2. The force exerted on 1 by 2 and the force exerted on 2 by 1 are internal forces. Their weights, however, are exerted by the earth, which is not in the box; the weights are therefor external forces.



As usual, let's start with a simple case, then generalize.

DERIVATION 7-2

Consider two objects, m_1 and m_2, each with its proper initial velocity, $\vec{v}_{1i}$and $\vec{v}_{2i}$. When the objects interact, they exert forces on each other that obey Newton's third law:

$$\vec{F}_{1,2} = -\vec{F}_{2,1} \ .$$

By the second law,

$$m_1\vec{a}_1 = -m_2\vec{a}_2 \ .$$

Careful here. Because the statement above is true instant by instant, it must also be true when averaged over the duration of the interaction between the masses, so we can write

$$m_1\vec{a}_{AVE\,1} = -m_2 a_{AVE\,2} \ .$$

Note that at this stage, we lose a lot of information about the forces involved. Remember the definition of the average acceleration and substitute:

$$m_1 \frac{\Delta\vec{v}_1}{\Delta t_1} = -m_2 \frac{\Delta\vec{v}_2}{\Delta t_2} \ .$$

We can make an argument using the third law that the two time intervals must be the same; if they were not, then there would be a time when one object would be exerting a force on the other while the other would not be exerting a force on the one.

$$m_1 \Delta\vec{v}_1 = -m_2 \Delta\vec{v}_2$$

$$m_1\vec{v}_{1f} - m_1\vec{v}_{1i} = -m_2\vec{v}_{2f} + m_2\vec{v}_{2i}$$

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \ .$$

At this point, we might recognize a relationship similar to one we saw in Section 6. The total momentum of the system before the interaction is equal to the total momentum of the system after the interaction. Momentum may well have been transferred from one object to the other, but the total momentum was conserved. Unlike energy, though, momentum does not change from one form to another, *e.g.*, from kinetic to potential.

For future reference, let's write this last expression as

$$m_1\vec{v}_{1f} + m_2\vec{v}_{2f} - m_1\vec{v}_{1i} - m_2\vec{v}_{2i} = \Delta\vec{p}_{TOTAL} = 0 \ .$$

Before we continue, a number of comments.

What if, in our derivation, there were an external force? Let's redo the work with an external force acting on mass 1 as an example. It would still be true that

$$\vec{F}_{1,2} = -\vec{F}_{2,1} \ .$$

But by the second law,

$$\vec{F}_{1,EXT} + \vec{F}_{1,2} = m_1\vec{a}_1 \ ,$$

so that

$$-\vec{F}_{1,EXT} + m_1\vec{a}_1 = -m_2 a_2 \ .$$

Following through to the end, we see that

$$-\vec{F}_{1,EXT} + m_1\vec{a}_{AVE\,1} = -m_2\vec{a}_{AVE\,2}$$

$$-\vec{F}_{1,EXT} + m_1 \frac{\Delta\vec{v}_1}{\Delta t_1} = -m_2 \frac{\Delta\vec{v}_2}{\Delta t_2}$$

$$-\vec{F}_{1,EXT}\Delta t + m_1\Delta\vec{v}_1 = -m_2\Delta\vec{v}_2$$

$$-\vec{F}_{1,EXT}\Delta t + m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \ .$$

$$\vec{F}_{1,EXT}\Delta t = \Delta\vec{p}_{TOTAL} \neq 0$$

and the total momentum is not conserved. We do not expect the total momentum of a system to be conserved if there are external forces.

However, there are two loopholes. The first, in analogy with the work energy theorem, is that momentum will still be conserved if the net external force is zero, that is, if the external forces happen to cancel to zero.[2] The other loophole is more common. Because we used vector notation, the derivation is valid for three dimensions. But remember that when two vectors are equal, their x, y, and z components are also independently equal. This means that the result can be written as three separate equations:

$$m_1 v_{1xi} + m_2 v_{2xi} = m_1 v_{1xf} + m_2 v_{2xf}$$

$$m_1 v_{1yi} + m_2 v_{2yi} = m_1 v_{1yf} + m_2 v_{2yf}$$

$$m_1 v_{1zi} + m_2 v_{2zi} = m_1 v_{1zf} + m_2 v_{2zf} \ .$$

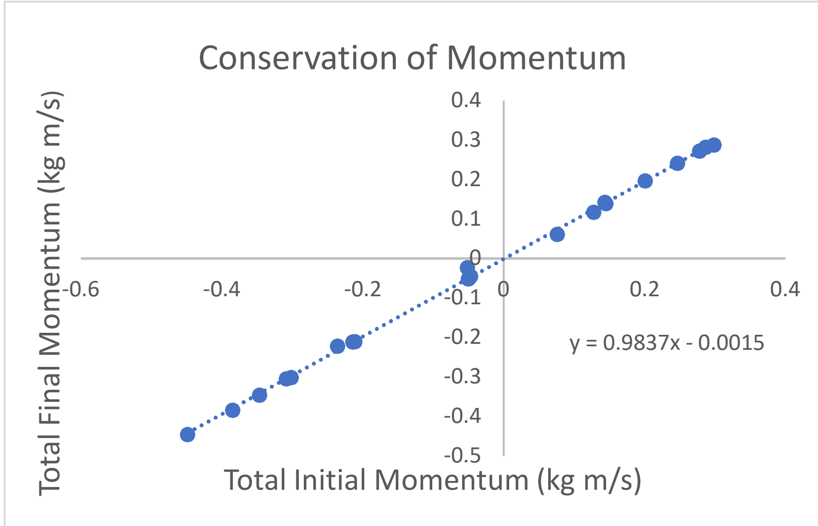
Suppose that there are some external forces in the x direction, but none in the y or z directions. Then, we can still use conservation of momentum in those two directions. As an example, think of two skydivers falling toward the earth. If the system is the two divers, there are external forces in the vertical direction (their weights), but none in the horizontal directions. Momentum will still be conserved horizontally in any collision the divers may suffer.

O.K., we are actually in a position to test the idea of conservation of momentum, and Newton's third law as well (remember that we skipped on that in Section 5) since our notion was based on Newton's second law (already tested) and the third law.

EXPERIMENT 7-1

[2] A simple analogous situation for work-energy might be to push a crate parallel to the surface on which it moves while balancing a kinetic frictional force.

Let's look at the results of an experiment to give us some confidence this is true. A system of two masses were placed on an airtrack to reduce friction (an external force) and collided together under different conditions. The vertical forces of weight and air from the track should not affect the horizontal motions. The velocities before and after were measured, and the total momentums before and after calculated and plotted. If conservation of momentum is true, a line of slope one through the origin should be seen. In these results, the intercept is very small compared to the values measured, and the slope is very close to one. This gives us some confidence that linear momentum is conserved, and indirectly, that the third law of motion is supported.



DERIVATION 7-3*

What if there are more than two masses in the system? Well, suppose that there are q masses. For each mass n, add up the k impulses acting on it.

$$\sum_{k} \vec{J}_{n,k} = \Delta\vec{p}_n$$

Add up these terms for all q masses:

$$\sum_{n=1}^{q} \sum_{k} \vec{J}_{n,k} = \sum_{n=1}^{q} \Delta\vec{p}_n = \Delta\vec{p}_{TOTAL} \, .$$

We can divide the impulses on the left into two categories, internal and external, in the same way we divided forces into conservative and non-conservative forces. Keep in mind that any object in the system won't exert a force on itself.

$$\sum_{n=1}^{q} \sum_{\substack{k=1 \\ k \neq n}}^{q} \vec{J}_{INT\ n,k} + \sum_{n=1,}^{q} \sum_{k} \vec{J}_{EXT\ n,k} = \Delta\vec{p}_{TOTAL} \, .$$

The second term on the left side is just the sum of the external impulses, so let's concentrate on the first term. Since $\vec{J} = \int \vec{F}(t)\, dt$,

$$\sum_{n=1}^{q}\sum_{\substack{k=1\\k\neq n}}^{q}\int \vec{F}_{n,k}(t)\,dt_{n,k} + \vec{J}_{EXT\ TOTAL} = \Delta\vec{p}_{TOTAL}\ .$$

Now, $dt_{n,k} = dt_{k,n}$ are time intervals during which objects n and k interact; by our third law argument above, these time intervals are equal and indeed occur simultaneously. From the third law, $\vec{F}_{n,k} = -\vec{F}_{k,n}$, and so we see that all of the terms in the summation cancel in pairs, leaving

$$\vec{J}_{EXT\ TOTAL} = \Delta\vec{p}_{TOTAL}\ .$$

If there are no external forces, then the total momentum of the system is conserved.

Brief Review

Let's review the three pictures:

- The velocity of an object will remain constant unless the object is acted on by a force, in which case $\vec{F}_{TOTAL} = m\vec{a}$.
- The kinetic energy of an object will remain constant unless the object has work performed on it, in which case $W_{TOTAL} = \Delta K$.
- The momentum of an object will remain constant unless the object is acted on by an impulse, in which case $\vec{J}_{TOTAL} = \Delta\vec{p}$.

These last two we can re-write for systems of objects:

- The total mechanical energy ($E = K + U$) of a system will remain constant unless the system has work performed on it by non-conservative forces, in which case $W_{NC} = \Delta E_{TOTAL}$.
- The total momentum of a system will remain constant unless the system is acted on by an external impulse, in which case $\vec{J}_{EXT} = \Delta\vec{p}_{TOTAL}$.

Collisions

As was noted above, conservation of momentum is particularly useful in analyzing collisions. You may have noticed that, in Derivation 7-X, the details of the forces acting between the objects disappeared, which is one of the strengths of this method. We don’t even need to know what kind of force acted on the objects! It is somewhat along the lines of having an object slide down along a curved frictionless surface; the details of the path were not necessary to find the speed of the object at the bottom.

We’re going to consider only the extremes of the spectrum of collisions. The easier of the two is the *completely inelastic collision,* one in which the objects stick together after the collision. Let's start by considering a simple situation in one dimension in which there are no external forces (that is, the only forces are those that each object exerts on the other):

EXAMPLE 7-2

An object of mass 5 kg is moving at 7 m/s along the +x axis when it strikes a stationary object of mass 3 kg. If they stick together, what is their common final velocity?

First of all, for a problem like this one, it's convenient to revert to the notation we used in Sections 2 through 5: a vector in the +x direction will carry a positive value, while one in the -x direction will carry a minus sign.

Assuming the two masses form a closed system, conservation of momentum seems appropriate.

$$m_1\vec{v}_{1xi} + m_2\vec{v}_{2xi} = m_1\vec{v}_{1xf} + m_2\vec{v}_{2xf} \ .$$

They have a common final velocity, or, if you prefer, it's as if they are now one object of mass $m_1 + m_2$ moving at velocity v_f.

$$m_1\vec{v}_{1xi} + m_2\vec{v}_{2xi} = (m_1 + m_2)\vec{v}_{xf} \ .$$

$$\vec{v}_{xf} = \frac{m_1\vec{v}_{1xi} + m_2\vec{v}_{2xi}}{m_1 + m_2} = \frac{5(+7) + 3(0)}{5+3} = +4.38\ \mathrm{m/s} \ .$$

EXAMPLE 7-3

An object of mass 12 kg is moving at 5 m/s along the +x axis has a *rear-end collision* with an object of mass 3 kg travelling at 4 m/s. If they stick together, what is their common final velocity?

First, what does rear-end collision mean? It's an expression from automobile collisions meaning that the two objects were moving in the same direction. A *head-on collision* would be one in which they were heading in the opposite directions.

Assuming the two masses form a closed system, conservation of momentum seems appropriate. Also, because they have a common final velocity,

$$m_1\vec{v}_{1xi} + m_2\vec{v}_{2xi} = (m_1 + m_2)\vec{v}_{xf} \ .$$

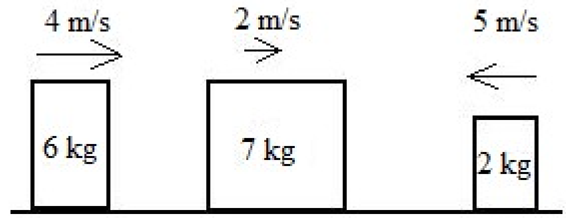
$$\vec{v}_{xf} = \frac{m_1\vec{v}_{1xi} + m_2\vec{v}_{2xi}}{m_1 + m_2} = \frac{12(+5) + 3(+4)}{12+3} = +4.8\ \mathrm{m/s} \ .$$

EXERCISE 7-1

An object of mass 6 kg is moving at 4 m/s along the +x axis has a *head-on collision* with an object of mass 3 kg travelling at 8 m/s. If they stick together, what is their common final velocity?

HOMEWORK 7-4

Three masses of 6 kg, 7 kg, and 2 kg move on a frictionless horizontal surface with initial speeds of 4 m/s, 2 m/s, and 5 m/s, respectively, as shown in the figure. If the masses all stick together after the collisions, what will be the final velocity of the combined mass?



HOMEWORK 7-5

Two railcars have a head-on collision, couple together, and stop dead. If Car A was moving four times as quickly as Car B was, and the total mass of both cars together is 90,000 kg, what are the masses of each car individually?

DISCUSSION 7-5

Consider the result of Exercise 7-1. We ended up with no momentum because we happened to start with no momentum. Sure, each object had some momentum of its own to start, but the total was zero. You may notice, however, that there is something else that we ended with zero of, but started with a positive amount of. What is it? Where did it go? Consider an automobile accident. What do the cars look like afterward and what was necessary to make them that way?

In Exercise 7-1, the objects started with 144 joules of kinetic energy, and ended with none. One of the characteristics of totally inelastic collisions is that kinetic energy is lost.

EXERCISE 7-2

Find the total initial and final kinetic energies in Example 7-x.

DERIVATION 7-4

Show that kinetic energy is always lost during the special case of a totally inelastic collision in one dimension when one of the objects is initially at rest. We've already shown that

$$\vec{v}_{xf} = \frac{m_1\vec{v}_{1xi} + m_2\vec{v}_{2xi}}{m_1 + m_2} \xrightarrow[m_2 \text{ starts at rest}]{} v_{xf} = \frac{m_1 v_{1xi}}{m_1 + m_2}.$$

Now we need to show that

$$\tfrac{1}{2}m_1v_{1xi}^2 + \tfrac{1}{2}m_2v_{2xi}^2 > \tfrac{1}{2}(m_1 + m_2)v_{xf}^2 \xrightarrow[m_2 \text{ starts at rest}]{}$$

$$\tfrac{1}{2}m_1 v_{1xi}^2 > \tfrac{1}{2}(m_1 + m_2)\left(\frac{m_1 v_{1xi}}{m_1 + m_2}\right)^2.$$

$$m_1 v_{1xi}^2 > \left(\frac{m_1}{m_1 + m_2}\right) m_1 v_{1xi}^2$$

This leads us to the true statement that

$$1 > \frac{m_1}{m_1 + m_2} ,$$

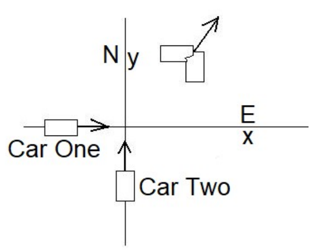
which is equivalent to saying that kinetic energy is always lost in this very special case.

DISCUSSION 7-6

The previous derivation was done for a very special case of one of the masses being initially at rest. After completing Section 7, you should be able to return here and make an argument that kinetic energy is lost in <u>any</u> totally inelastic collision regardless of the initial motions of the two masses.

Let's examine a particular situation. Suppose that a bullet is fired into a block of wood. The bullet penetrates a given distance into the block, and the block of course moves a bit in the direction the bullet was moving. The bullet applied a force to the block and the block applied an equal though opposite force to the bullet. The bullet did positive work on the block, and the block did negative work on the bullet. However, the displacements of each object were not the same during this process, and so more negative work was done on the bullet than positive work done on the block. As a result, kinetic energy was lost.

EXAMPLE 7-4



Let's try a two-dimensional example. Suppose you are an insurance accident investigator. Two cars collided at an icy intersection as shown, and the wreckage moved off at an angle 59° north of east. You know that Car One (1500 kg) was moving eastward at 30 m/s just before the collision, because it was caught on a speed camera. The question is, how quickly was Car Two (2000 kg) moving?

We'll let the system comprise the cars. The road is icy, or frictionless, so there are no external horizontal forces. The vertical normal forces and weights will not prohibit conservation of momentum in the horizontal directions. Let east be the +x direction and north be the +y direction. Use conservation of momentum separately in each direction.

$$x:\ m_1 v_{1xi} + m_2 v_{2xi} = (m_1 + m_2)\, v_{xf}$$

$$y:\ m_1 v_{1yi} + m_2 v_{2yi} = (m_1 + m_2)\, v_{yf}$$

In this solution, v_{1yi} and v_{2xi} are both zero, and $v_{xf} = v_f \cos(\theta)$ and $v_{yf} = v_f \sin(\theta)$.

$$m_1 v_{1xi} = (m_1 + m_2)\, v_f \cos(\theta)$$

$$m_2 v_{2yi} = (m_1 + m_2)\, v_f \sin(\theta)$$

Divide the second equation by the first to obtain

$$\frac{m_2 v_{2yi}}{m_1 v_{1xi}} = \tan(\theta)\ .$$

Then,

$$v_{2yi} = \frac{m_1}{m_2}\, v_{1xi}\, \tan(\theta) = \frac{1500}{2000}(30)\tan(59^o) = 37.4\ \mathrm{m/s}\,.$$

Now, let's consider a *totally elastic collision*, by which we mean no kinetic energy is lost during the collision (although, it can be transferred from one object to the other). Think of the objects as having springs on them; instead of kinetic energy being used to deform the objects, some kinetic energy is stored as potential energy, then re-released as kinetic. For reasons that will be discussed later, this derivation will be applicable to problems in one dimension only.

DERIVATION 7-5

We will write one equation representing conservation of momentum (in one dimension only) and another representing the fact that the total kinetic energy is the same before and after the interaction.

$$m_1\vec{v}_{1xi} + m_2\vec{v}_{2xi} = m_1\vec{v}_{1xf} + m_2\vec{v}_{2xf}$$

$$\tfrac{1}{2}m_1v_{1xi}^2 + \tfrac{1}{2}m_2v_{2xi}^2 = \tfrac{1}{2}m_1v_{1xf}^2 + \tfrac{1}{2}m_2v_{2xf}^2$$

Typically, we are given the masses and initial velocities and are asked to find the final velocities. Since we have two independent equations and two unknowns, we should be good. One solution should be obvious: $v_{1xi} = v_{x1f}$ and $v_{2xi} = v_{x2f}$; the equations require merely that K and **p** be conserved, which is certainly the case if no collision actually occurs. However, finding the other, more interesting, solution requires about two pages of effort. So, what we're going to do is what physicists often do when a problem is too difficult; we'll look at a special case. Here, we'll simplify the problem to require that mass two is initially at rest. Of course, the results we obtain will be valid for only that situation. Our two equations become

$$m_1\vec{v}_{1xi} = m_1\vec{v}_{1xf} + m_2\vec{v}_{2xf}$$

$$\tfrac{1}{2}m_1v_{1xi}^2 = \tfrac{1}{2}m_1v_{1xf}^2 + \tfrac{1}{2}m_2v_{2xf}^2$$

Reverting to our Section 2 notation, this first equation can be rewritten as

$$m_2 v_{2xf} = m_1 v_{1xi} - m_1 v_{1xf} = m_1(v_{1xi} - v_{1xf})$$

and the second as

$$\tfrac{1}{2} m_2 v_{2xf}^2 = \tfrac{1}{2} m_1 v_{1xi}^2 - \tfrac{1}{2} m_1 v_{1xf}^2 = \tfrac{1}{2} m_1 \,(v_{1xi} - v_{1xf})(v_{1xi} + v_{1xf}) \;.$$

Dividing the second equation by the first and multiplying through by two results in

$$v_{2xf} = v_{1xi} + v_{1xf} \;,$$

which we substitute into the original momentum equation.

$$m_1 v_{1xi} = m_1 v_{1xf} + m_2(v_{1xi} + v_{1xf})$$

Solving for the final velocity of mass one gives us

$$v_{1xf} = \frac{m_1 - m_2}{m_1 + m_2} v_{1xi} \;.$$

If instead, we substitute to solve for the final velocity of mass two, we get

$$v_{2xf} = \frac{2m_1}{m_1 + m_2} v_{1xi} \;.$$

Once again, remember that these solutions are only valid if one mass had been initially at rest, the collision was totally elastic, and motion was restricted to one dimension. To be clear, you should label whichever mass was not initially moving as mass two. If you encounter this type of problem in a homework or exam question, you may move directly to these two relationships as your starting point.

EXAMPLE 7-4

An object of mass 12 kg moving at 5 m/s along the +x axis has a totally elastic collision with a stationary object of mass 3 kg. What are their final velocities?

As allowed above, we will start with the two relationships we derived. The solution becomes 'plug-and-chug.'

$$v_{1xf} = \frac{m_1 - m_2}{m_1 + m_2} v_{1xi} = \frac{12 - 3}{12 + 3} 5 = 3\ \mathrm{m/s} \,.$$

$$v_{2xf} = \frac{2m_1}{m_1 + m_2} v_{1xi} = \frac{2(12)}{12\ 3} 5 = 8\ \mathrm{m/s} \,.$$

EXERCISE 7-3

An object of mass 7 kg is moving at 10 m/s along the +x axis has a totally elastic rear-end collision with an object of mass 4 kg travelling at 3 m/s. What are their final velocities?

What if neither mass had been at rest? Well, we could go back and re-do the derivation with the two extra terms, but here is a neat trick: we can make use of the material of Section 4 (relative motion) and pick a new *frame of reference* (indicated below by a *prime*) in which mass 2 is initially at rest, calculate the final velocities in that frame, then convert back to the original frame.

Let's use the previous Exercise as an example. For the observer who described the problem, Mass 1 is moving in the +x direction at 10 m/s before it hits Mass 2 moving the same way at +3 m/s. If we were passengers riding alongside Mass 2, we would of course think that Mass 2 is stationary and see Mass 1 approaching us from behind at 7 m/s. From our point of view, the relationships derived above would be perfectly O.K. to use. Then, we need only calculate what the original observer sees.

EXAMPLE 7-5

I like to keep track of this process with a chart. It also makes the process somewhat mechanical, and thereby less susceptible to mistakes. The information for each mass runs horizontally in the rows. The first column contains the original values for each mass's initial velocity. The third column contains the initial velocities in the new frame of reference; the initial velocity of Mass 2 here must be zero. The second column is the process that changes the values. We ask, what must be done to M_2's initial velocity to make it zero? In this case, we must subtract 3 m/s.

	$v_{initial}$	convert to new frame in which $v_{2xi}' = 0$	v_o'
M_1	+10 m/s		
M_2	+ 3 m/s	Subtract 3 m/s	0 m/s

Of course, if we subtract 3m/s from M_2's velocity, we must do the same for M_1:

	$v_{initial}$	convert to new frame in which $v_{2xi}' = 0$	v_o'
M_1	+10 m/s	Subtract 3 m/s	+ 7 m/s
M_2	+3 m/s	Subtract 3 m/s	0 m/s

Now we have a problem we can solve. Use the relationships derived, we can find the final velocities in the new frame of reference.

	v_i	convert	v_i'	Find v_f'

M_1	+10 m/s	-3	+7 m/s	$v'_{1xf} = \frac{m_1 - m_2}{m_1 + m_2} v'_{1xi} = \frac{7-4}{7+4} 7 = 1.91\ \mathrm{m/s}$.
M_2	+3 m/s	-3	0 m/s	$v'_{2xf} = \frac{2m_1}{m_1 + m_2} v'_{1xi} = \frac{2(7)}{7+4} 7 = 8.91\ \mathrm{m/s}$.

We're not done, because we need to convert back to the original frame. We do that by reversing the transformation that we did previously, in this example, by adding 3 m/s to the results.

	v_i	convert	v_i'	Find v_f'	convert back to original frame by reversing the previous transformation	v_f
M_1	+10 m/s	-3	+7 m/s	$v'_{1xf} = \frac{m_1 - m_2}{m_1 + m_2} v'_{1xi} = \frac{7-4}{7+4} 7 = 1.91\ \mathrm{m/s}$.	+3	4.91 m/s
M_2	+3 m/s	-3	0 m/s	$v'_{2xf} = \frac{2m_1}{m_1 + m_2} v'_{1xi} = \frac{2(7)}{7+4} 7 = 8.91\ \mathrm{m/s}$.	+3	11.91 m/s

Before I give you an exercise to try, let's do another short derivation. To be honest, I have never found the result of this to be useful, except as a quick check of my results for the chart solution. I'll show you what I mean in a moment.

DERIVATION 7-6*

Here is an additional interesting derivation for a totally elastic collision. Here, we do not need to assume that m_2 is initially at rest. That is, the result is valid for any one-dimensional totally elastic collision.

We start with the conditions for conservation of momentum and kinetic energy:

$$m_1 \vec{v}_{1xi} + m_2 \vec{v}_{2xi} = m_1 \vec{v}_{1xf} + m_2 \vec{v}_{2xf}$$

$$\tfrac{1}{2} m_1 v_{1xi}^2 + \tfrac{1}{2} m_2 v_{2xi}^2 = \tfrac{1}{2} m_1 v_{1xf}^2 + \tfrac{1}{2} m_2 v_{2xf}^2 .$$

Let's re-arrange each:

$$m_1(v_{1xi} - v_{1xf}) = m_2(v_{2xf} - v_{2xi})$$

$$m_1(v_{1xi} - v_{1xf})(v_{1xi} + v_{1xf}) = m_2(v_{2xf} - v_{2xi})(v_{2xf} + v_{2xi})$$

Dividing the second equation by the first leaves

$$v_{1xi} + v_{1xf} = v_{2xf} + v_{2xi} \quad .$$

So if the chart solution was done correctly, we find that the sums of the masses' initial and final velocities should be the same. For example, 10 + 4.91 = 3 + 11.91. You can use this as a quick check on your answers. An agreement won't guarantee you're correct, but a failure will tell you if you're wrong.

HOMEWORK 7-6

A 10 kg object initially moving to the right at 20 m/s makes a totally elastic head on collision with a 15 kg object which was initially moving to the left at 5 m/s. Find the final velocities of each object.

HOMEWORK 7-7

A 10 kg object initially moving to the right at 20 m/s has a totally elastic rear-end collision with a 15 kg object which was initially moving to the right at 5 m/s. Find the final velocities of each object.

JUSTIFICATION OF ASSUMPTIONS*

In the method discussed above, that is changing to a new frame of reference to solve our problem, we assumed that if momentum and kinetic energy are conserved in one frame, that they are conserved in the other frame. We need to justify those assumptions. It's not too difficult for momentum. Let's bite the bullet and do it for three dimensional collisions. For the original frame, we can rewrite the momentum equation as

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} - m_1\vec{v}_{1f} - m_2\vec{v}_{2f} = 0 \quad .$$

Let $\vec{u}$ be the velocity of the first frame relative to the second frame. Then in that new frame we ask if,

$$m_1(\vec{v}_{1i} + \vec{u}) + m_2(\vec{v}_{2i} + \vec{u}) - m_1(\vec{v}_{1f} + \vec{u}) - m_2(\vec{v}_{2f} + \vec{u}) = 0 \quad .$$

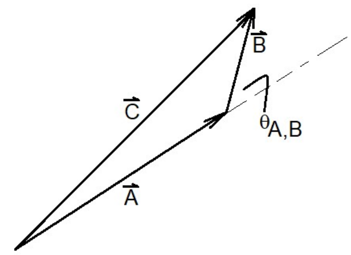
Re-arranging a bit results in

$$(m_1\vec{v}_{1i} + m_2\vec{v}_{2i} - m_1\vec{v}_{1f} - m_2\vec{v}_{2f}) + (m_1 + m_2 - m_1 - m_2)\,\vec{u} = 0 \quad .$$

The first term is zero from our knowledge of the initial frame, and the second term is clearly zero, and so momentum is conserved in the new frame. Since there was no restriction put on

$\vec{u}$, momentum is conserved in every possible frame if it is conserved in any one frame, regardless of the type of collision.

Kinetic energy is a bit more difficult because we deal with the objects' speeds, not their velocities. Let's review a bit. Suppose we add two vectors, $\vec{A}$ and $\vec{B}$, that are not in the same (or opposite) directions and want to know the magnitude of the sum, $|\vec{A}+\vec{B}|$. Draw $\vec{A}$, $\vec{B}$, and their sum $\vec{C}$ so as to form a triangle. The *law of cosines* tells us that[3]



$$C^2 = A^2 + B^2 + 2AB\cos\theta_{A,B} = A^2 + B^2 + 2\,\vec{A}\cdot\vec{B}\,.$$

Note that this is a general statement that reduces to the Pythagorean theorem when theta is 90°. Now, in our original frame of reference, let's assume that kinetic energy is conserved during the collision. We can write

$$\tfrac{1}{2}m_1v_{1i}^2 + \tfrac{1}{2}m_2v_{2i}^2 - \tfrac{1}{2}m_1v_{1f}^2 - \tfrac{1}{2}m_2v_{2f}^2 = 0\ .$$

In the new frame, we'd like to know if

$$\tfrac{1}{2}m_1|\vec{v}_{1i}+\vec{u}|^2 + \tfrac{1}{2}m_2|\vec{v}_{2i}+\vec{u}|^2 - \tfrac{1}{2}m_1|\vec{v}_{1f}+\vec{u}|^2 - \tfrac{1}{2}m_2|\vec{v}_{2f}+\vec{u}|^2 = 0\ .$$

After multiplying it all out and re-arranging a bit,

$$\left(\tfrac{1}{2}m_1v_{1i}^2 + \tfrac{1}{2}m_2v_{2i}^2 - \tfrac{1}{2}m_1v_{1f}^2 - \tfrac{1}{2}m_2v_{2f}^2\right) + (m_1\vec{v}_{1i} + m_2\vec{v}_{2i} - m_1\vec{v}_{1f} + m_2\vec{v}_{2f})\cdot\vec{u}$$
$$+\ \tfrac{1}{2}(m_1 + m_2 - m_1 - m_2)\,u^2 = 0\ .$$

The contents of the first and second sets of parentheses are zero from our knowledge of the original frame of reference (K_{TOTAL} and $\vec{p}_{TOTAL}$ were conserved), and that of the third is clearly zero, and so if the kinetic energy is conserved in any frame, then it is conserved in every frame. We can return to Derivation 7-4 and generalize the result: if kinetic energy is lost during a collision in one frame, some is lost in any frame, including any in which both objects were initially moving.

This is going to be an important point in Physics Three.

While we're here, let's think about a couple of other considerations that students have asked about over the years. What about impulse and work in different frames? Suppose a mass has a force acting on it for a given duration of time. What is the force in a new frame of reference?

$$\vec{F}\Delta t = m(\vec{v}_f - \vec{v}_i) = m(\vec{v}_f - \vec{v}_i) + m(\vec{u} - \vec{u}) = m\big((\vec{v}_f + \vec{u}) - (\vec{v}_i + \vec{u})\big)$$

[3] The angle is defined differently here than is usual. It is the exterior angle rather than the interior angle, which leads to the difference in sign.

$$= m(\vec{v}_f' - \vec{v}_i') = \vec{F}'\Delta t \ .$$

So, a force of a certain magnitude in one frame of reference has the same magnitude in any other frame of reference, as does the impulse.[4] Knowing that, what can we say about the work done by a force in two different frames? We certainly expect that the work could be different because the displacements could be different. The work-energy theorem in the original frame will be

$$W = \vec{F} \cdot \Delta\vec{r} = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) \ .$$

In a new frame moving at velocity $\vec{u}$ with respect to the original frame, we have that

$$W' = \vec{F}' \cdot \Delta\vec{r}' = \vec{F} \cdot (\Delta\vec{r} + \vec{u}\,\Delta t) = \vec{F} \cdot \Delta\vec{r} + \vec{F} \cdot \vec{u}\,\Delta t = W + \vec{F} \cdot \vec{u}\,\Delta t \ .$$

So, fun fact, if the new frame is moving perpendicularly to the force, the work in each frame is the same. Continuing,

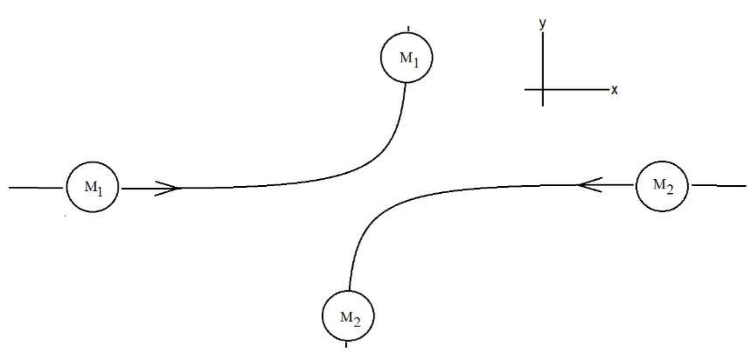
$$W' = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) + \left(\vec{F}\,\Delta t\right) \cdot \vec{u} = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) + m(\vec{v}_f - \vec{v}_i) \cdot \vec{u}$$
$$= \tfrac{1}{2}m\left(v_f^2 - v_i^2 + 2(\vec{v}_f - \vec{v}_i) \cdot \vec{u} + u^2 - u^2\right) =$$

$$\tfrac{1}{2}m(|v_f + u|^2 - |v_i + u|^2) = \tfrac{1}{2}m\left(v_f'^2 - v_i'^2\right) = \Delta K' \ .$$

So, in any frame, the work done in that frame is the change in kinetic energy in that frame, but certainly not necessarily the same change as in another frame, as we expected.

ADMONITION*

When we discussed totally inelastic collisions, we made the point that we could treat a three-dimensional problem as three separate one-dimensional problems. You were warned, however, not to treat totally elastic problems that way. Let's discuss briefly why we can not simply use the chart method above three times, one for each direction.



The derivation that resulted in those relationships for the final velocities required the total kinetic energy to be conserved. To split the solution up into three separate parts would require that the contributions to the kinetic energy due to motion in any one of the directions would also need to be

[4] That is, if the time intervals in each frame are the same, which is a characteristic of Galilean transformations. The problem comes about in relativistic transformations, which we'll discuss in Semester Three.

conserved, which is a much stricter requirement. Here is an illustration of a two-dimensional situation in which this strict requirement would not be met. Consider two masses heading toward each other that undergo a glancing collision, as shown. Before the interaction, there is kinetic energy due to the motions in the x-direction and none due to the y-motion. After, however, the situation is reversed. So the kinetic energy overall is conserved, but it is not conserved independently in each direction. Consequently, the relationships we have been using are not valid for anything other than a one-dimensional collision.

DISCUSSION 7-6

Does this mean that it is impossible to solve two-dimensional totally elastic collision problems? What is the general rule for solving algebraic systems of equations?

We can solve any of these problems so long as we have enough information and patience, although the solution may be difficult algebraically. Let's look at two situations.

Consider two masses that collide totally inelastically in three dimensions. Given the masses and the initial velocities, can we find the final velocities?

$$m_1 v_{1xi} + m_2 v_{2xi} = (m_1 + m_2)\, v_{xf}$$

$$m_1 v_{1yi} + m_2 v_{2yi} = (m_1 + m_2)\, v_{yf}$$

$$m_1 v_{1zi} + m_2 v_{2zi} = (m_1 + m_2)\, v_{zf}$$

Three equations and three unknowns; we're good. In fact, we did a two-dimensional example earlier.

Consider two masses that collide totally elastically in three dimensions. Given the masses and the initial velocities, can we find the final velocities?

$$m_1 v_{1xi} + m_2 v_{2xi} = m_1 v_{1xf} + m_2 v_{2xf}$$

$$m_1 v_{1yi} + m_2 v_{2yi} = m_1 v_{1yf} + m_2 v_{2yf}$$

$$m_1 v_{1zi} + m_2 v_{2zi} = m_1 v_{1zf} + m_2 v_{2zf}$$

$$\begin{aligned}\tfrac{1}{2}m_1\left(v_{1xi}^2 + v_{1yi}^2 + v_{1zi}^2\right) + \tfrac{1}{2}m_2\left(v_{2xi}^2 + v_{2yi}^2 + v_{2zi}^2\right) \\ = \tfrac{1}{2}m_1\left(v_{1xf}^2 + v_{1yf}^2 + v_{1zf}^2\right) + \tfrac{1}{2}m_2\left(v_{2xf}^2 + v_{2yf}^2 + v_{2zf}^2\right)\end{aligned}$$

Here, unfortunately, we have six unknowns, but only four independent equations. We need more information.

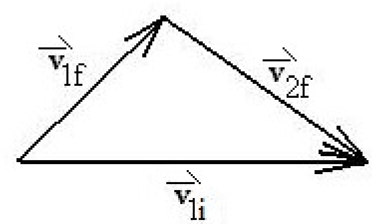
EXAMPLE 7-6

Let's look at a very special case, that of the masses being equal and mass two initially at rest. This can be made into a two-dimensional problem, since all of the momentum vectors line in a plane (you had a question on Sample Exam One along these lines). We'll write the equations for conservation of momentum (in vector form) and kinetic energy.

$$m_1\vec{v}_{1i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \quad \rightarrow \quad \vec{v}_{1i} = \vec{v}_{1f} + \vec{v}_{2f}$$

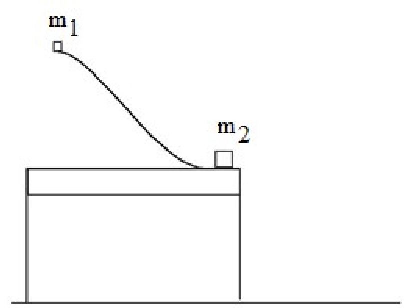
$$\frac{1}{2}m_1v_{1i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 \quad \rightarrow \quad v_{1i}^2 = v_{1f}^2 + v_{2f}^2$$

The first equation says we can make a triangle with the velocity vectors like the one at right, and the second, which looks a lot like the Pythagorean theorem, is only going to be true if the triangle is a right triangle, so that $\vec{v}_{1f}$ and $\vec{v}_{2f}$ are at right angles to one another, a nice result. Notice however, that this does not give us the actual directions or magnitudes of the velocities; to know those, we need more information.



EXERCISE 7-4

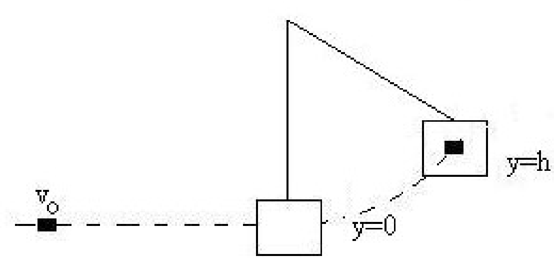
Here is a nice synthesis problem. It requires you to choose which of the three 'pictures' we have developed to use in each section. Keep in mind that the three pictures are essentially identical, but that one may be much more convenient to use than the other two in a given situation.



A 0.2 kg block (m_1) is released from rest at the top of a frictionless, curved track 1.5 meters above the top of a 1.1 meter high table. At the bottom of the track, where it is horizontal, this mass collides elastically with a 0.8 kg mass (m_2) that is initially at rest. How far from the base of the table does the 0.8 kg mass land?

HOMEWORK 7-8

The *ballistic pendulum* is a device used to measure the muzzle velocity of a bullet. A block of wood of mass M is suspended by a string from the ceiling, and the bullet of mass m is fired horizontally into it. As the block moves backward with the embedded bullet, it swings upward to some maximum height.[5] If the bullet has mass 2 g, the block has mass 2.5



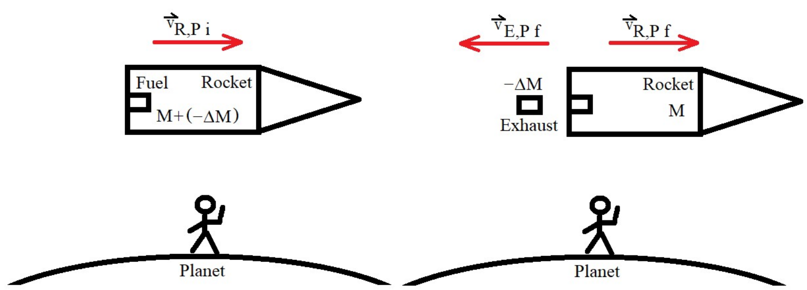
[5] When I was much younger, I taught at a school out west where this was actually done with a .22 in lab class.

kg, and the block/bullet combination rises through a vertical distance of 6 cm, find the initial speed v_o of the bullet.

Outside the Safety Zone*

Calculus-based textbooks often wrap this section up with *the rocket equation*. But we won't need calculus because we've already solved the pertinent equation in Section 5. So, let's concentrate on the physics instead.

DERIVATION 7-6*



Consider a rocket of mass M travelling past a planet at velocity $\vec{v}_{R,P}$. At that time, it has just ejected some exhaust at a velocity $\vec{v}_{E,R}$ relative to itself and at velocity $\vec{v}_{E,P} = \vec{v}_{E,R} + \vec{v}_{R,P}$ relative to the planet. Next comes what I think is the really tricky part: the mass of the rocket changes by amount *d*M, which is a negative quantity, but the mass of the ejected exhaust is must be positive, so -*d*M. Let's make use of conservation of momentum, while ignoring gravity, from just before fuel ejection to just after. During that process, the rocket's velocity increases by amount $d\vec{v}_{R,P}$. Let's make 'to the right' in the figure be positive.

$$(\mathrm{M} + (-d\mathrm{M}))\mathrm{v}_{\mathrm{R,Pi}} = \mathrm{M}\mathrm{v}_{\mathrm{R,Pf}} + (-d\mathrm{M})\mathrm{v}_{\mathrm{E,Pf}}$$

$$(\mathrm{M} + (-d\mathrm{M}))\mathrm{v}_{\mathrm{R,P\,i}} = \mathrm{M}(\mathrm{v}_{\mathrm{R,P\,i}} + d\mathrm{v}_{\mathrm{R,P}}) + (-d\mathrm{M})(\mathrm{v}_{\mathrm{R,P\,i}} - \mathrm{v}_{\mathrm{E,R}}) \quad .$$

We can cancel out quite a few terms, and let's drop the 'initial' subscript:

$$0 = \mathrm{M}\, d\mathrm{v}_{\mathrm{R,P}} + d\mathrm{M}\, \mathrm{v}_{\mathrm{E,R}} \ ,$$

$$\frac{\mathrm{dM}}{\mathrm{M}} = -\frac{1}{\mathrm{v}_{\mathrm{E,R}}}\, \mathrm{dv}_{\mathrm{R,P}} \ ,$$

$$\int_{\mathrm{M_o}}^{\mathrm{M}} \frac{d\mathrm{M}}{\mathrm{M}} = -\frac{1}{\mathrm{v}_{\mathrm{E,R}}} \int_{\mathrm{v}_{\mathrm{R,P\,i}}}^{\mathrm{v}_{\mathrm{R,P}}} d\mathrm{v}_{\mathrm{R,P}}$$

$$\ln\frac{\mathrm{M}}{\mathrm{M_o}} = -\frac{1}{\mathrm{v}_{\mathrm{E,R}}}(\mathrm{v}_{\mathrm{R,P}} - \mathrm{v}_{\mathrm{R,P\,i}}) \quad .$$

Let's clean it up a bit by dropping the R,P and setting $v_{R,P\,i}$ to just v_o.

$$\ln\frac{M}{M_o} = -\frac{1}{v_{E,R}}(v - v_o)\ .$$

From here, we can go several ways:

$$v = v_o + v_{E,R}\ \ln\left(\frac{M_o}{M}\right)\quad \text{or}\quad M = M_o e^{\frac{-(v-v_o)}{v_{E,R}}}\ .$$

Here, M_o is the mass of the rocket and all of its fuel at the start of the problem when its speed is v_o, and M is the mass of the rocket and its unexpended fuel when the speed is v. It looks a bit strange perhaps because there is no explicit time dependence.

EXAMPLE 7-6*

The spaceship HMCSS Clark is 'at rest' and fully fueled at Space Station TALC. Her mass is 2×10^7 kg, with all but 2 per cent of it fuel. Her engine expels exhaust at 3 km/s. What maximum speed can she attain relative to Station TALC?

Starting with our previous result, re-arranging, and setting $M = 0.02\ M_o$,

$$v = v_o + v_{E,R} \ln\left(\frac{M_o}{M}\right) = 0 + 3000\ \ln\left(\frac{1}{0.02}\right) = 11{,}736\ \mathrm{m/s}\ .$$

HOMEWORK 7-9*

In a severe pinch, Space Force decides to utilize *Lenkflugkörper NG* missiles to defend earth from the Jovian attackers during a deep space battle. The missiles themselves have a mass of 3 kg and contain an additional 22 kg of fuel with an exhaust velocity of 465 m/s. They must reach a speed of 700 m/s relative to the launching space vessel. What is the largest payload that could be attached to one?

EXERCISE 7-1 Solution

Assuming the two masses form a closed system, conservation of momentum seems appropriate. Also, because they have a common final velocity,

$$m_1\vec{v}_{1xi} + m_2\vec{v}_{2xi} = (m_1 + m_2)\vec{v}_{xf}\ .$$

$$\vec{v}_{xf} = \frac{m_1\vec{v}_{1xi} + m_2\vec{v}_{2xi}}{m_1 + m_2} = \frac{6(+4) + 3(-8)}{6 + 3} = 0\ \mathrm{m/s}\,.$$

EXERCISE 7-2 Solution

$$K_i = \frac{1}{2}m_1 v_{1i}^2 + 0 = \frac{1}{2}5\,(7^2) = 122.5\text{ J}$$

$$K_f = \frac{1}{2}(m_1 + m_2)v_f^2 = \frac{1}{2}8\,(4.38^2) = 76.7\text{ J}$$

EXERCISE 7-3 Solution

Are your answers 3 m/s and 8 m/s? Did you confirm that the problem meets the criteria for using the two relationships derived in class? Does it?

EXERCISE 7-4 Solution

This problem has three parts. There is m_1 sliding down the incline. There is the collision. There is the trajectory of m_2 as it travels toward the floor. Each of these is best treated with one of the three pictures we have discussed.

As m_1 slides down the ramp, it is acted on by a normal force and by its weight. There is no friction. We have no details about the actual shape of the ramp, and apparently we do not care how much time it takes for the mass to reach the bottom of the ramp. This looks like a job for work-energy!

$W_N = 0$ (the normal force is always perpendicular to the path)
W_g – conservative

$$0 = \frac{1}{2}m_1\,v_f^2 - \underbrace{\frac{1}{2}m_1\,v_i^2}_{\text{starts from rest}} + gm_1y_f - gm_1y_i$$

Let's put y = 0 at the foot of the table. We want to find v_f, the speed of m_1 just before the collision.

$$0 = \frac{1}{2}v_f^2 + gy_f - gy_i$$

$$v_f = \sqrt{2g(y_i - y_f)} = \sqrt{2(10)(2.6 - 1.1)} = 4.47\text{ m/s}\,.$$

The second part of the problem is a collision, and that screams for conservation of momentum. During the interaction between the masses, they are moving horizontally with no external horizontal forces acting on them. There are vertical external forces (the weights and the normal forces from the ramp), but that doesn't preclude conservation of momentum in the horizontal direction. Because it's a totally elastic collision in one dimension with mass 2 initially at rest, we can jump right to the relationships we derived for just such a situation:

$$v_{2xf} = \frac{2m_1}{m_1 + m_2}v_{1xi} = \frac{2(0.2)}{0.2 + 0.8}4.47 = 1.79\text{ m/s}\,.$$

The last part of the problem is projectile motion. Let's put the origin at the foot of the table, with +x to the right and +y upward. Our inventory is

$x_i = 0$ m

$x_f = ?$ ←

$v_{xi} = +1.79$ m/s
$v_{xf} = +1.79$ m/s
$a_x = 0$ m/s^2
$t = ?$

Since there is not enough information on the x-side, we need to look to the y-side and try our 80% Rule.

$y_i = 1.1$ m
$y_f = 0$ m
$v_{yi} = 0$ m/s (the ball was travelling horizontally as it left the table)
$v_{yf} = ?$
$a_y = -10$ m/s^2 (we chose upward to be positive)
$t = ?$

KEq. 3:

$$y_f = y_i + v_{yi}t + \tfrac{1}{2}a_y t^2$$

This will become a quadratic equation in t. Inserting the numbers and re-arranging to the standard format leaves us with

$$(5)t^2 + (0)t + (-1.1) = 0\,,$$

which, it turns out, we can solve directly:

$$t = \pm\sqrt{\frac{1.1}{5}} = +0.47 \text{ seconds} \;.$$

Take this back to the x-side to find x_f.

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 = 0 + 1.79(0.47) + 0(0.47^2) = 0.84 \text{ m} \;.$$