28.78. A long, straight, solid cylinder, oriented with its axis in the z-direction, carries a current whose current density is $\vec{J}$. The current density, although symmetrical about the cylinder axis, is not constant and varies according to the relationship

$$\vec{J} = \left(\frac{b}{r}\right)e^{(r-a)/\delta}\hat{k} \qquad \text{for } r \le a$$

$$= \mathbf{0} \qquad \text{for } r \ge a$$

where the radius of the cylinder is $a = 5.00$ cm, r is the radial distance from the cylinder axis, b is a constant equal to 600 A/m, and δ is a constant equal to 2.50 cm. (a) Let I_0 be the total current passing through the entire cross section of the wire. Obtain an expression for I_0 in terms of b, δ, and a. Evaluate your expression to obtain a numerical value for I_0. (b) Using Ampere's law, derive an expression for the magnetic field $\vec{B}$ in the region $r \ge a$. Express your answer in terms of I_0 rather than b. (c) Obtain an expression for the current I contained in a circular cross section of radius $r \le a$ and centered at the cylinder axis. Express your answer in terms of I_0 rather than b. (d) Using Ampere's law, derive an expression for the magnetic field $\vec{B}$ in the region $r \le a$. (e) Evaluate the magnitude of the magnetic field at $r = \delta$, $r = a$, and $r = 2a$.