

3-2)

We need to find the difference in potential between the spheres. Let's assume that the inner has uniform charge +Q while the outer has charge -Q. This is not really that important in that we're using Gauss's law for E:

$$E(r) = \frac{k_e Q}{r^2} .$$

Then,

$$V = - \int_{R_1}^{R_2} k_e Q r^{-2} dr = k_e Q r^{-1} \Big|_{R_1}^{R_2} = k_e Q \left(\frac{1}{R_2} - \frac{1}{R_1} \right) .$$

Let's flip that around to make it positive:

$$V = k_e Q \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = k_e Q \frac{R_2 - R_1}{R_1 R_2} .$$

Now, the definition of capacitance is

$$C = \frac{Q}{V} = \frac{Q}{k_e Q \frac{R_2 - R_1}{R_1 R_2}} = \frac{1}{k_e} \frac{R_1 R_2}{R_2 - R_1} .$$

Note that, as we make R_2 grow to infinity (our first example in class), this result becomes the expected R_1/k_e .