

6-9 Soln)

$$\begin{aligned} V_{L MAX} &= \chi_L I_{MAX} = \omega L \frac{\mathcal{E}_{MAX}}{Z} = \mathcal{E}_{MAX} L \frac{\omega}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \\ &= \mathcal{E}_{MAX} L \omega \left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \right)^{-1/2} \end{aligned}$$

Set $dV_{L MAX}/d\omega = 0$ and solve for omega.

$$\begin{aligned} \mathcal{E}_{MAX} L \left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \right)^{-1/2} \\ - \mathcal{E}_{MAX} L \omega \left(\frac{-1}{2} \right) \left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \right)^{-\frac{3}{2}} 2 \left(\omega L - \frac{1}{\omega C}\right) \left(L + \frac{1}{\omega^2 C}\right) = 0 \\ \left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \right)^{-1/2} - \omega \left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \right)^{-\frac{3}{2}} \left(\omega L - \frac{1}{\omega C}\right) \left(L + \frac{1}{\omega^2 C}\right) = 0 \end{aligned}$$

One solution is

$$\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \right)^{-1/2} = 0 ,$$

which in turn corresponds to $\omega = 0$ or $\omega = \infty$, which in turn correspond to $V_{L MAX} = 0$. Let's clean up some of the mess:

$$\begin{aligned} 1 - \omega \left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \right)^{-1} \left(\omega L - \frac{1}{\omega C}\right) \left(L + \frac{1}{\omega^2 C}\right) &= 0 \\ 1 = \omega \left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \right)^{-1} \left(\omega L - \frac{1}{\omega C}\right) \left(L + \frac{1}{\omega^2 C}\right) \\ 1 = \left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \right)^{-1} \left(\omega^2 L^2 - \frac{1}{\omega^2 C^2}\right) \\ R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 &= \omega^2 L^2 - \frac{1}{\omega^2 C^2} \\ R^2 + \omega^2 L^2 - 2 \frac{L}{C} + \frac{1}{\omega^2 C^2} &= \omega^2 L^2 - \frac{1}{\omega^2 C^2} \end{aligned}$$

$$R^2 + -2\frac{L}{C} + \frac{2}{\omega^2 C^2} = 0$$

$$\frac{1}{\omega^2}=LC-\frac{R^2}{2}$$

$$\omega=\left(LC-\frac{R^2C^2}{2}\right)^{-1/2}$$