PM-6 Soln)

We can use Bernoulli's equation:

$$P_1 + \tfrac{1}{2}Dv_1^2 + gDy_1 = P_2 + \tfrac{1}{2}Dv_2^2 + gDy_2 \quad .$$

We'll look at the top of the container, and just outside the hole. The pressure at these points should be the same at atmospheric (although, to be AR, slightly higher at the hole than at the top). Let y = 0 at the bottom, so y_1 = H and y_2 = 0.

$$\tfrac{1}{2}Dv_1^2 + gDH = \tfrac{1}{2}Dv_2^2 \quad .$$

Now, if R>>r, then the speed of the water at the top will be approximately zero (the water level doesn't fall very quickly) and we have

$$gDH = \tfrac{1}{2}Dv_2^2 \quad .$$

$$v_2 = \sqrt{2gH} \quad .$$

If not, then we'll also need the continuity equation:

$$A_1v_1 = A_2v_2$$

so that

$$\tfrac{1}{2}D\left(\frac{\pi r^2}{\pi R^2}\right)v_2^2 + gDH = \tfrac{1}{2}Dv_2^2 \quad ,$$

$$\left(\frac{r}{R}\right)^2 v_2^2 + 2gH = v_2^2 \quad ,$$

$$\left[1 - \left(\frac{r}{R}\right)^2\right] v_2^2 = 2gH \quad ,$$

$$v_2 = \sqrt{\frac{2gH}{1 - \left(\frac{r}{R}\right)^2}} \quad .$$