Section 2-5 - Magnetism: Effects & Causes

The Magnetic Field

Magnetism is an effect which has been known for several millennia. It was found that pieces of a certain kind of rock, called *lodestone* (Fe_3O_4), can attract and/or repel one another, as well as other bits of metals. These rocks were common in a province near Turkey named Magnesia, and so the effect is called *magnetism*. When suspended and allowed to rotate freely, these rocks always orient themselves the same way; the end which points toward the north is called a *north-seeking-pole (*or just north pole), and the other end is the *south-seeking-pole* (south pole). This allows the construction of which important navigation tool?
Answer

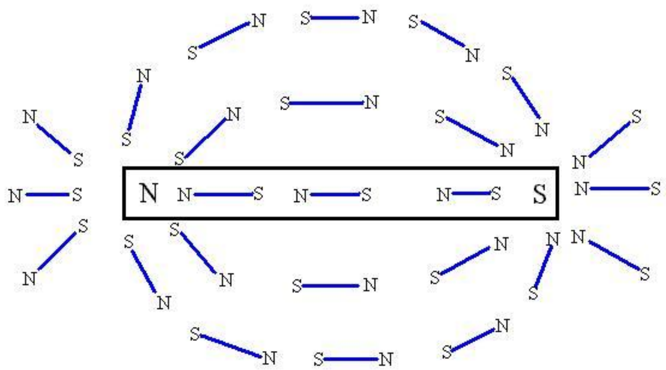
We find that magnetic poles always come in pairs, unlike electric charges; whenever one type of pole is present, the other type must be somewhere near. There is a theory which postulates the existence of a *magnetic monopole*, but none has been seen to date.

We can make artificial lodestones from iron (and other materials), which we call *magnets*. Investigating the interactions between magnets, we find that:
Like poles repel,
Unlike poles attract.

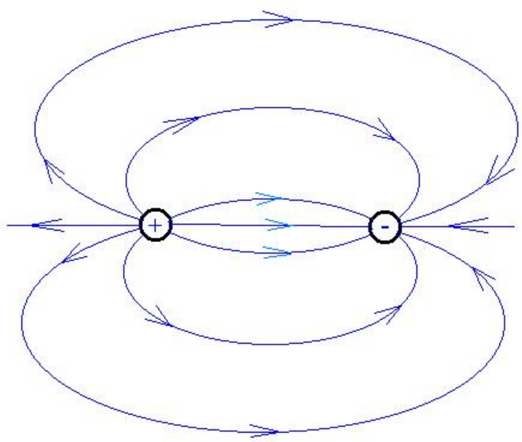
In order for a compass to work, what must be at (or near) the earth's Geographic North Pole?

Answer

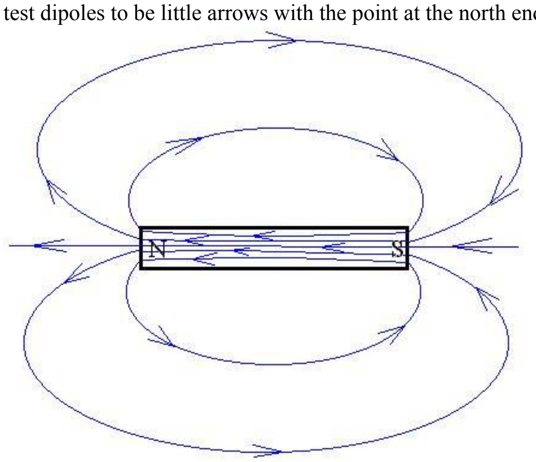
This is very similar to the rules that govern the interactions of electric charges, so perhaps some analogies can be drawn. For example, can we model the interactions of two poles with a *magnetic field*, somewhat similar to the electric field? Since there are no magnetic monopoles to use as magnetic test charges, we will instead use magnetic dipoles as our test objects. We saw in a previous section that electric dipoles will tend to align with the electric field. Let's place some small test compasses in the vicinity of a larger magnet:



It seems that we could indeed think of the magnetic dipoles as aligning with a magnetic field, which resembles fairly closely the electric field for two equal but opposite charges:



So let's define the *magnetic induction field* (symbol **B**, unit the *Tesla*; see Note, below) and arbitrarily choose its direction as follows: consider the test dipoles to be little arrows with the point at the north end that point in the direction of the magnetic field:



We see that the field runs from the north pole to the south pole, except inside the magnet, where it runs from south pole to north pole. This is somewhat different from the situation with charges, in which the electric field ran from positive charges to negative charges. Here, we see that the magnetic field lines instead form closed loops.

Gauss's Law for Magnetism

We defined the electric flux through a surface to be

$\Phi_E = \int E_{perp}\, dA.$

That is, we look at each little piece of surface area dA and multiply that area by the perpendicular component of the electric field at that point, counting the product as positive if the field points from the inside to the outside of the surface, and negative otherwise. We found that, if we sum this quantity up over a closed surface, that the the result is proportional to the charge enclosed by the surface:

$\Phi_E = \oiint E_{perp}\, dA = 4\pi k_e q_{enclosed}.$

Perhaps there is a similar relationship for the magnetic field. We define the magnetic flux (unit, the *Weber*) through a closed surface as:

$\Phi_M = \oiint B_{perp}\, dA$

Consider a closed surface and calculate the total magnetic flux through it. As for the electric flux, let Φ_M be positive if the field points from the inside to the outside of the surface, and negative if it points from the outside to the inside. Remember that an alternate picture of the electric flux was the net number of electric field lines that pass through the surface. What, then, is the total magnetic flux through a closed surface equal to?

Answer

This result will be correct so long as there really is no such thing as a magnetic monopole.

Interactions between Currents and Magnetic Fields

We performed a demonstration. We used a horseshoe magnet to produce a magetic field in the vicinity of a current carrying wire, and saw that a force was exerted on the wire. We saw that the magnitude of the force depended on the size of the current, the strength of the field, and the length of the wire. If we had done very careful quantitative measurements, we would have seen that:

$F_M \sim B$

$F_M \sim I$

$F_M \sim L$.
In addition, we saw that the force was directed at right angles to the direction of the current and to the direction of the B-field. We suspected that we might be able to predict the direction of the force using the Right-Hand-Rule (RHR), if we take the current direction as the first vector and the field direction as the second. If true, there should be a dependence on the relative orientation of the current and the field, and this could be found to be true by doing careful quantitative measurements. So,
$F_M = ILB\sin\theta_{I,B}$ (RHR), or $\mathbf{F} = L\,\mathbf{I} \times \mathbf{B}$.
That is, point your forefinger in the direction of the current, your middle finger in the direction of the B-field, and your thumb should be in the direction of the force.

Now consider the effects of a magnetic field on individual charged particles. A *cathode ray tube* (CRT) such as those in televisions shoot a stream of electrons toward a screen, which is covered with a fluorescent material which glows as the electrons strike. We orient the magnetic field in different ways in the vicinity of the electrom beam, and find a similar deflection. Suppose that the beam is coming out toward you and that the magnetic field (from a horseshoe magnet) is pointing toward your left. Based on the previous discussion, which way would you expect the force on the electrons to be directed, as evidenced by the their deflection?

Answer

Did you get the wrong answer? Why?
Answer

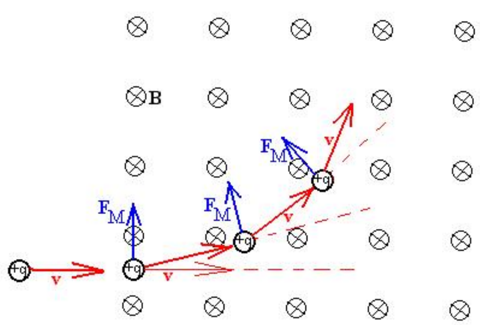
Is there an angle dependence of the force? We tested this by turning the magnet around and saw that the deflection becomes smaller as the B-field becomes more parallel (or anti-parallel) to the velocity.

Let's guess a relationship for the force:
We expect that the force is proportional to the charge on the particles: $F \sim q$. This could be verified by using alpha particles instead of electrons or protons.
We could vary the speed of the particles, ands we would find that $F \sim v$.
We can add magnets to vary B, and we find that $F \sim B$.
Also, the angular dependence $F \sim \sin\theta_{v,B}$, so that
$F_M = qvB\sin\theta_{v,B}$ (RHR), or $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$

Note: Some define the magnetic field line as that path along which a charged particle can move and experience no magnetic force.

Is this consistent with our previous result? In a naive way, we can write that, for a wire,
$F_M = ILB\sin\theta_{I,B}$ (RHR) $= [q/t]LB\sin\theta_{I,B}$ (RHR) $= q[L/t]B\sin\theta_{I,B}$ (RHR) $= qvB\sin\theta_{I,B}$ (RHR) $= qvB\sin\theta_{v,B}$ (RHR) or $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$.
where we understand that the direction of the product of q and v is in the direction of the current (that is, if q is positive, v and I and parallel, but if q is negative, then qv and I are parallel).

Let's consider a special case of a charged particle (mass m and charge +q) entering a region of uniform magnetic field (**B**) so that initially, **v** and **B** are perpendicular.



There will be a magnetic force acting on the particle, $F_M = qvB$ (remember that we stipulated that $\theta_{v,B}$ is 90^o). This force will be direcred as shown, and will deflect the path of the particle from its original direction to the one shown. Once it has arrived at its 'new position' with its new velocity, there will still be a magnetic force, but that force will be directed at a slightly different angle, as shown. This causes another deflection and the particle moves to the third position shown, *et c*.
What can we say about the speed of the particle?

Answer

Keeping in mind that this process is much smoother, and that the path taken is not really polygonal, can we guess what shape trajectory the particle will take?
Answer

What kind of force is required to keep an object moving in such a trajectory?

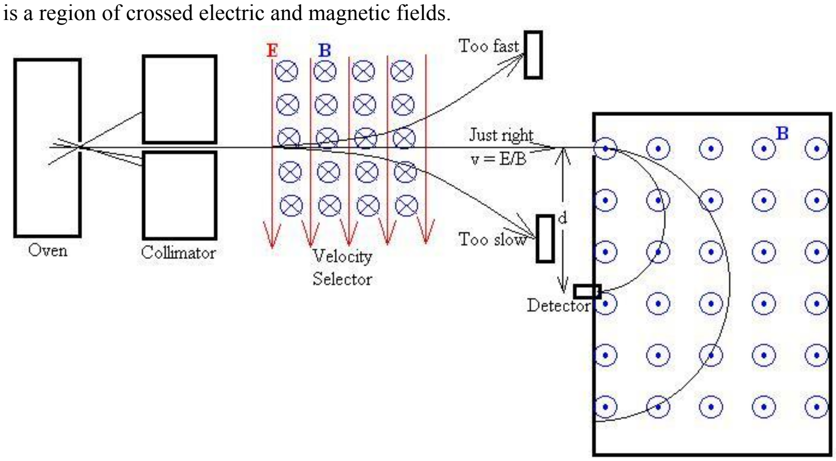
Answer

What provides that force in this situation?
Answer

So, we have that
$F_M = F_C$
$qvB = mv^2/r$
$qB = mv/r$
$r = mv/qB$.

Let's look at this same situation in a slightly different way to obtain an interesting result:
$F_M = F_C$
$qvB = mr\omega^2$
$qr\omega B = mr\omega^2$
$qB = m\omega$
$\omega = qB/m$.
That is, the angular velocity (or, more interestingly and equivalently, the frequency) of a charged particle in a magnetic field is independent of the energy (speed) of the particle!

This result is the foundation for several interesting devices, notably the *mass spectrometer*. Suppose that we wish to identify a material from its molecular mass. We place the material in an oven which vaporizes it and spews it out at high speed. Unfortunately, there is a distribution of speeds (the Maxwell distribution). After passing through various collimators to produce a well defined beam of particles, they molecules are directed through an *electron stripper*, which removes one (or more) electrons. The now charged particles travel through a *velocity selector*, which is a region of crossed electric and magnetic fields.



There is an electric force qE on the particle toward the bottom of the page and a magnetic force qvB directed toward the top of the page (check it!). The electric force is the same regardless of the speed of the particle, but the magnetic force is proportional to the speed of the particles. For fast particles, F_M is greater than F_E, and the particles will be deflected upward. For slow particles, F_E is greater than F_M, and the particles are deflected downward. However, for particles with just the right speed, the forces cancel and the particles travel in a straight line. This will occur when
$F_E = F_M$
$qE = qvB$ (assume q is not zero)
$E = vB$
$v = E/B$.
Now, the particles enter the main spectrometer, a region of uniform B-field, and their paths are bent into circular arcs as shown. Since all particles now have the same speed, and (we hope) the same charge, and they are experiencing the same B, we find that the radius of each orbit is proportional to the mass of the particle:
$r = [v/qB]\ m$.
By running a detector along one edge of the chamber and recording the number of particles arriving as a function of distance d (=2r), a spectrum of the masses of the particles can be taken and the materials identified.
$m = [qB_{SPECT}B_{SELECT}/2E_{SELECT}]d$.
Often there are complications: for example, some particles will be doubly or triply charged, giving rise to extraneous lines at m/2 and m/3, and the molecules are often broken into fragments which also register as peaks in the spectrum.

This process was initially used to separate uranium early in the wartime effort to construct the atomic bomb. There are several isotopes of uranium, the most common being U-235 and U-238. U-235 is useful for bombs, while U-238 is useful in power reactors, but will squelch the necessary fast chain reaction in a bomb. They can not be separated chemically, since they have the same electronic structure. A method was developed to use the mass spectrometer to separate the isotopes, but the production rate was eventually deemed to be too slow.

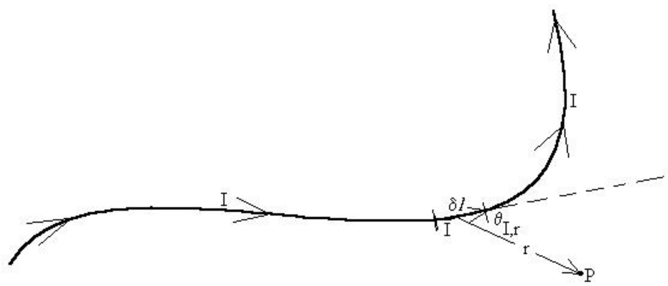
Consider these questions:
What shape path will a charged particle take when injected into a uniform electric field with its initial velocity not perpendicular to the field?

Answer

What shape path will a charged particle take when injected into a uniform magnetic field with its initial velocity not perpendicular to the field?
Answer

Biot-Savart Law

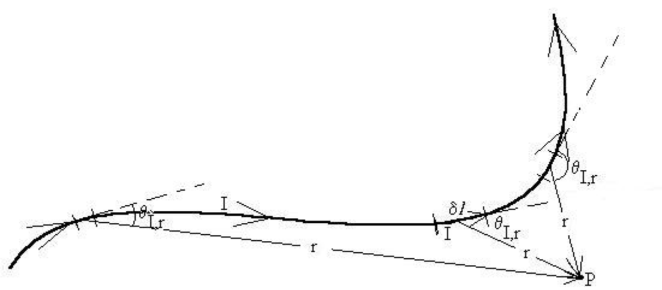
We have seen that magnetic field can have an effect on electric currents. Now we shall discuss the observation that currents can cause magnetic fields. Previously, we had assumed that these fields were caused by special rocks (which we can produce artificially, it's true). Later, we will present a naive model to reconcile these two sources of B-field. How do we know that currents cause B-fields? Weirdly enough, it is from a lecture demonstration gone bad. Oersted had wanted to show that currents in wires do not affect compass needles, but he learned the hard way that they do, as the compass swung around in front of his audience.
We won't attempt to justify the following result in anyway, other that to say that it is based on careful experimentation and observation. Consider a current of arbitrary shape; we wish to find the magnetic field caused by this current at point P.



Consider a short piece of the current of length δl_i, which produces a contribution $\delta\mathbf{B}_i$ to the field at point P. We find that the magnitude of δB_i is given by

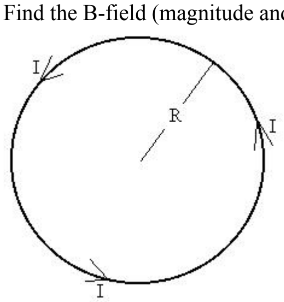
$\delta B = k_M\, I\, \delta l\, \sin\theta_{I,r} / r^2$,

where k_M is a constant to make the units work out right ($k_M = 10^{-7}$ Tm/A, exactly), r is the distance from the current element and the point P at which we are finding the field, and $\theta_{I,r}$ is the angle between the direction of the current and the direction to the point P. We find that the direction of this contribution to the B-field is given by a Right-Hand-Rule: the index finger points in the direction of the current, the middle finger points toward the point at which the field is being found, and the thumb points in the direction of $\delta\mathbf{B}$. In this example, the contribution from the curremt element shown is into the page.
To find the total field at P, we need to add up all the contributions from each little piece of the current, keeping in mind that this must be <u>vector</u> addition. Also, r_i and $\theta_{I,r}$ may well be different for each current element:



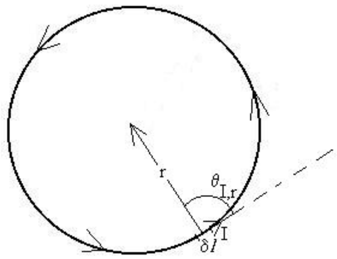
Even then, that only gets the B-field at one spot; the calculation must be repeated for any other spot. In some special cases, we may find that the contributions all point in the same direction, in which case the vector addition simplifies to algebraic addition.

Example:
Find the B-field (magnitude and direction) at the centre of a circular loop of radius R carrying current I as shown.



Answer

Consider a short bit of the loop of length δl_i. The distance r from this piece of current to the point at which we are finding the B-field is R, and the angle between the current direction and the direction to the centre is 90^o. From the Right-Hand-Rule, the field this piece of current causes points out of the page.



What's more, all these things are true for <u>any</u> of the current elements. So, the vector sum we must do reduces to an algebraic sum:

$B = \int \delta B = \int k_M I \, \delta l \, (\sin\theta_{I,r})/r^2$,

But we said that the current is the same in each element, r = R for each element, and θ is 90^o (so sinθ = 1) for each element, so we can factor these constants out of the integral:

$B = [k_M I/R^2] \int \delta l$.

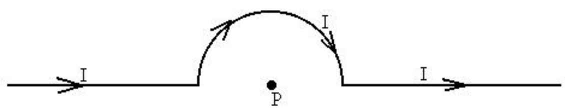
The sum all around the loop of all the small lengths is the circumference of the circle, $2\pi R$, so,

$B = [k_M I/R^2]\, 2\pi R = 2\pi k_M I/R$.

Now, we will make a change of the constant, just as we changed $4\pi k_e$ into $1/\varepsilon_o$. We let $\mu_o = 4\pi k_M$, so that the result becomes

$B_{centre} = \mu_o I/2R$.

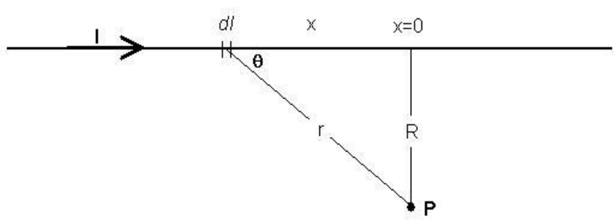
Consider this current arrangement:



The straight portions of the wire extend out to infinity and the curved jag is a semi-circle of radius R. Find the magnetic field at P.

Answer

Here is another example:
Infinitely long, straight wire with current I



Consider the P point shown. The small length of wire *dl* will produce a contribution to the magnetic field at P

$\delta B = k_M I \, \delta l \sin\theta_{I,r}/r^2$

To change variables for integration, let $r = [R^2 + x^2]^{1/2}$, $dl = dx$, and $\sin\theta = R/r = R/[R^2 + x^2]^{1/2}$.

$\delta B = k_M I \, \delta x \, R \, [R^2 + x^2]^{-3/2}$

Since each contribution *d*B will, by the RHR, point into the page, the total field at P is the simple sum of all the contributions (the magnitude of the sum is the sum of the magnitudes):

$B_P = \int_{-INF}^{+INF} k_M I \, \delta x \, R \, [R^2 + x^2]^{-3/2} = k_M I R \int \delta x \, [R^2 + x^2]^{-3/2} = k_M I R \left[x/R^2[R^2 + x^2]^{1/2} \Big|_{-INF}^{+INF}\right] = 2k_M I/R = \mu_o I/2\pi R$.

Ampère's Law

Ampère's Law can be shown to be identical to the Biot-Savart Law (we won't do so). From an operational point of view, Ampère's law is to magnetism what Gauss's law is for electricity; for sufficiently nice symmetries, we can use it to determine the magnetic field caused by current distributions.

Suppose that we have a current which is producing a magnetic field B. Let's draw an imaginary closed loop in space in the vicinity of the current. Break the loop up into many very short lengths, δl. As we follow the path along the loop, we look at the component of the magnetic field which is parallel (or anti-parallel) to the loop and multiply it by the length of the little bit of the loop:
$B_{||}\, dl$.
This is analogous to choosing an imaginary surface and looking at the perpendicular component of the electric field in Gauss's law:
$E_{perp}\, dA$
Now, let's add up all these terms all the way around the loop; if the B-field component is in the same direction in which we are traversing the loop, we'll call the term positive and if the component is opposite to the direction we are going, we'll call it negative. This is again analogous to calling the flux positive if the E-field points from the inside of the gaussian surface to the outside, and calling it negative if the E-field points from the outside in. When the sum is complete, the result will be proportional to the net current passing through the loop:

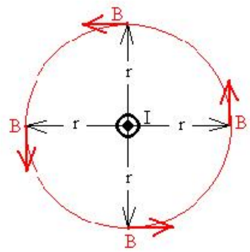
$$\oint B_{||}\, dl = \mu_o I_{enclosed}$$

or more explicitly mathematically:

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_o I_{enclosed}$$

Just as for Gauss's Law, this is always true, but it may be useful for finding B only in certain symmetric situations. In Section 2-1, we tried to find surfaces for which either the E-field was along the surface (so that $E_{perp} = 0$) or if not so, then exactly perpendicular and constant in magnitude. Similar hopes and dreams run wild here: we'd like to choose loop paths such that either B is perpendicular to the path (and so there is no contribution to the sum) or if not, then preferably along the path and constant in magnitude.

Consider the example of an infinite, straight, current-carrying wire. First, we need to at least consider the Biot-Savart Law (and the RHR) to get an idea of what the B-field looks like:



It seems clear that the B-field will form circular loops around the wire, as shown. Furthermore, symmetry arguments can be made that the magnitude of the field at points at any given distance r from the wire will be the same. It seems that a good choice for our imaginary loop might be a circle of radius r which lies along one of the field loops. In that case, we write that

$$\oint B_{||}\, dl = \mu_o I_{enclosed}$$

Since B is parallel to the loop at all points, we can replace $B_{||}$ with plain ole B:

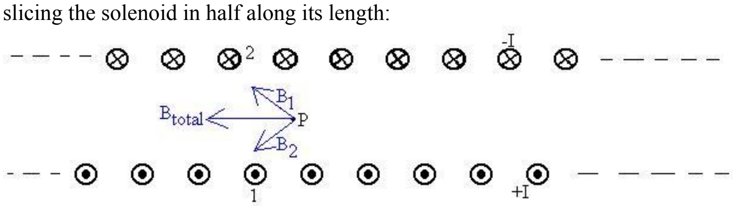
$$\oint B\, dl = \mu_o I_{enclosed}$$

Also, since we argued that the magnitude of B is the same everywhere on the loop, we can pull it out of the integral:

$$B \oint dl = \mu_o I_{enclosed}$$

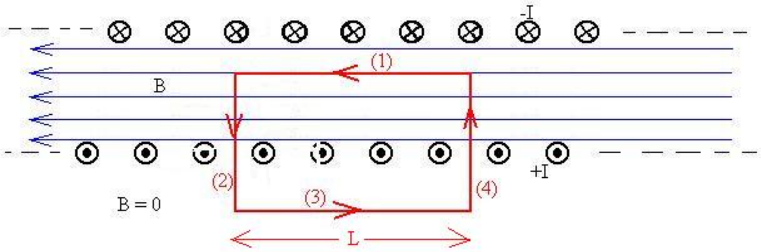
The integral has been reduced to the length of the loop, *i.e.*, the circumference of the circle:
$B\, 2\pi r = \mu_o I_{enclosed}$.
$B = [\mu_o I_{enclosed}]/2\pi r$,
and in this case, $I_{enclosed}$ is just the given current I, so
$B = \mu_o I/2\pi r$.
It's acceptable to have r in the answer since r corresponds to the real distance from the wire at which we want to find the field.

Consider the infinitely long *solenoid*, which is a wire coiled up along a cylindrical form, somewhat like a 'slinky.' Let's draw this schematically by slicing the solenoid in half along its length:



The lower vectors represent the current coming under the bottom of the solenoid, while the upper vectors represent the current rolling back over the top of the solenoid. We pick a point P inside the solenoid on its central axis at which to find the field. Consider one little bit of the current, here labeled '1.' The B-field from that bit of current is in the direction shown and is labeled B_1. Likewise for a bit of current exactly opposite it and the same distance from point P, labeled '2.' We see that the net field is directed to the left, since the vertical components will cancel. This will be true for all such pairs we pick, so the net field is to the left. Also, we can say that the magnitude of B will be the same all along that central axis, since the solenoid is infinite in length and we can shift it to the right or left any amount and the system should appear to be the same. This is true if the points we look at are on the central axis of the solenoid; what if we look at a point off axis? We shall contend that the B-field is to the left everywhere inside the solenoid, based again on symmetry arguments.

What about the field outside of the solenoid? We contend that the B-field there is to the right and very weak, weak enough to neglect. Remember that B-field lines from closed loops, so the lines running inside the solenoid towards the left end have to loop back on the outside to the right end. Since the solenoid is infinitely long, we might expect that the lines are very spread out from each other on the 'return trip,' and since the strength of the field is related to how closely the lines are spaced, we can say that the field is very weak.
Let's choose a rectangular path of length L along which to calculate our sum:



Label the segments of the path as shown. The sum can be written as:

$\oint B\, dl = \mu_o I_{enclosed} = \int_{(1)} B_{\|}\, dl + \int_{(2)} B_{\|}\, dl + \int_{(3)} B_{\|}\, dl + \int_{(4)} B_{\|}\, dl = \mu_o I_{enclosed}$.

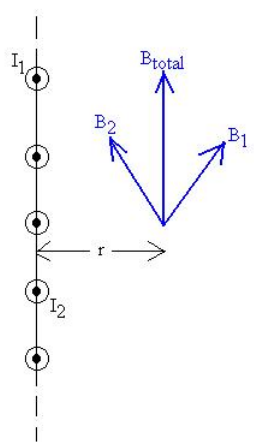
Along (1), B is parallel to the path and has constant magnitude, so $B_{\|} = B$, and so it can be factored out of the integral to give B $\int_{(1)} dl = BL$.

Along (2), B is either zero or it is perpendicular to the path, so the contribution is zero.
Along (3), B is zero, so the contribution is zero.
Along (4), B is either zero or it is perpendicular to the path, so the contribution is zero.
So, we're left with
$BL = \mu_o I_{enclosed}$.
Now, work on the right hand side of the relationship. $I_{enclosed}$ depends on how many turns of the solenoid are enclosed; let's just say N turns. Then,
$BL = \mu_o NI$.
$B = \mu_o NI/L$.
Now, we have a problem. When we discussed Gauss's Law, we pointed out that there should be no quantities in our final answer that are dependent on the dimensions of our imaginary surface. In this result, we have two quantities that depend on the size of our imaginary loop, namely the length L and the number of turns enclosed, N. What is the resolution to this paradox?

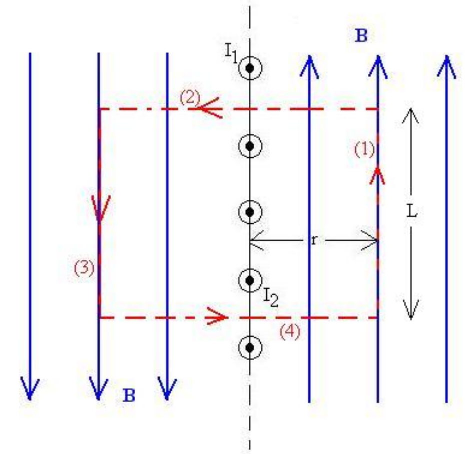
Answer

Let n = N/L, so that the result becomes
$B = \mu_o nI$.
Now, note that this result does not depend on where we placed part (1) of the rectangular loop, so that the magnitude of B is the same across the cross-sectional area of the solenoid. This means that solenoids are very useful for producing extremely uniform magnetic fields, such as necessary for NMR experiments.

Lastly, let's look at the example of a flat, infinite sheet of current. The current at any spot is directed parallel to some given axis (in the figure, it's all out of the page). Since the total amount of current is infinite (so long as it's not zero), we should define a *current density*, the amount of current *per* unit length: $\mathcal{J} = I/L$. This is not the usual current density J, the current *per* unit area. Look at some point a distance r from the sheet and try to determine the appearance of the B-field.



Consider two currents, I_1 and I_2, each producing a contribution to the total field; we see that the vertical components add and the horizontal components cancel. In addition, since we can slide the sheet up or down (or for that matter, into or out of the page) some distance and it will still look the same, we can say that the magnitude of the B-field is the same value at any point a given distance r from the sheet. What's more, the field on the other side of the sheet at distance r will have that same magnitude, although we can see quickly with the Right-Hand-Rule that the direction is reversed.



Let's choose a rectangularly shaped loop about which to sum. Since we as yet have no idea how the B-field strength varies with distance from the sheet, we'll draw the loop symmetrically as shown; then at least the field magnitudes along parts (1) and (3) will be the same.
So,

$$\oint B_{||}\, dl = \mu_o I_{enclosed} = \int_{(1)} B_{||}\, dl + \int_{(2)} B_{||}\, dl + \int_{(3)} B_{||}\, dl + \int_{(4)} B_{||}\, dl = \mu_o I_{enclosed}.$$

Along (1), B is parallel to the path and has constant magnitude, so $B_{||} = B$ and it can be factored out of the integral to give $B \int_{(1)} dl = BL$.

Along (2), B is perpendicular to the path, so the contribution to the sum is zero.

Along (3), B is parallel to the path and has constant magnitude, so $B_{||} = B$ and it can be factored out of the integral to give $B \int_{(3)} dl = BL$.

Along (4), B is perpendicular to the path, so the contribution to the sum is zero.
So, we're left with
$2BL = \mu_o I_{enclosed}$.
Now, work on the right hand side of the relationship.

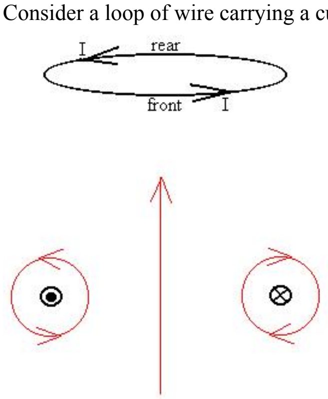
$I_{enclosed} = \mathcal{J} L$, so

$2BL = \mu_o \mathcal{J} L$.

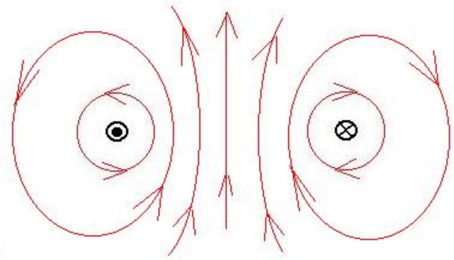
$B = \mu_o \mathcal{J}/2$.
Note that the answer is independent of r, the distance from the sheet.

Magnetic Moment

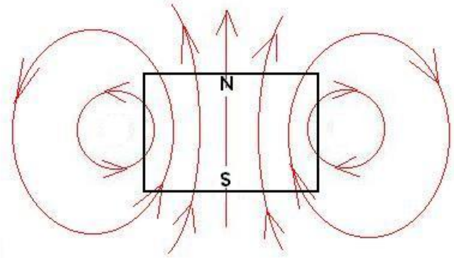
Consider a loop of wire carrying a current I (since this is a three dimensional situation, we'll label the front and rear parts of the loop):



In the lower half of the figure, we've sliced the loop in half with a plane parallel to that of this page, so that only the two bits passing through that plane are now shown. Let's get an idea of what the B-field produced by this loop of current looks like. We previously used the Biot-Savart Law to show that the field points along the axis of the loop points as shown, and when we are near the wire, we can consider the portion nearest us to behave much like a straight bit and ignore the more distant bits, thus producing the field as shown here. Let's also fill in the field imaginatively in the other regions, keeping in mind that there is probably a smooth transition from one region to the other:



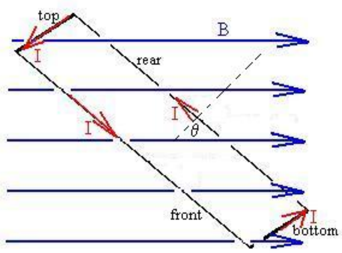
Now, does this field arrangement remind us of anything? Perhaps the magnetic field caused by a bar magnet?



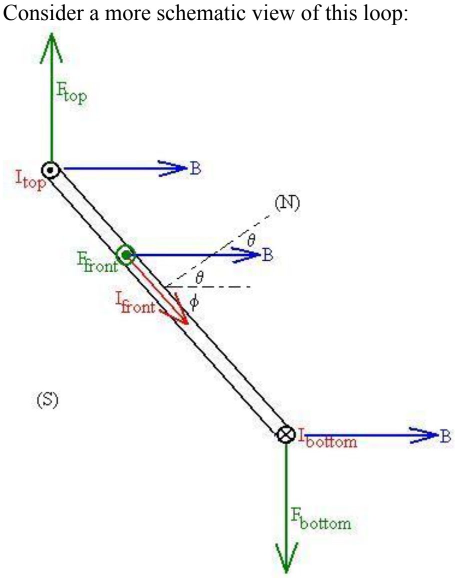
So, we should expect a loop of current to behave like a bar magnet or compass needle, and we should be able to predict which side of the loop will act like a north pole (short-hand Right-Hand-Rule: Curl fingers of RH in direction of the current, and the thumb will be where the north pole is). We did a quick demonstration of this and saw that, indeed, the current loop was attracted and repelled by a bar magnet as expected.

Why does a bar magnet act like a current loop? See below.

Let's consider a rectangular loop (length *l* and width *w*) carrying current I, in a uniform magnetid field, B, the plane of which is inclined to the direction of the field by some angle θ, which we will measure between the magnetic field and a line perpendicular to the plane of the loop:



Consider a more schematic view of this loop:



In this figure, the current running up the back leg is not shown. From the previous discussion, we might well expect that the current will cause to form a north pole somewhere up and to the right of the loop (and a south pole down and to the left) and that, as a result, the loop will begin to swing clockwise to align itself with the magnetic field. Let's examine this in closer detail:
The loop comprises four legs or straight sections, each carrying current I, in a magnetic field B. There will therefor be a magnetic force on each

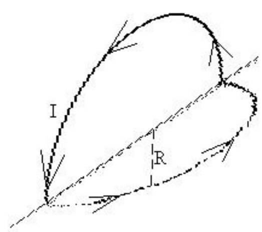
leg,
$F_M = ILB \sin\theta_{I,B}$ (RHR):

$F_{top} = IwB \sin 90^o = IwB$ (up),
$F_{front} = IlB \sin\phi$ (out of the page), ϕ is the complement of our original θ.
$F_{bottom} = IwB \sin 90^o = IwB$ (down),
$F_{rear} = IlB \sin(180^o - \phi) = IwB \sin\phi$ (into page).

Now, the net force on the loop is clearly zero, but what about the torque? On the front and rear sides, the net torque will be zero, since, for every bit of current on the front side experiencing a magnetic force, there is an oppositely directed current bit experiencing a force in the other direction which as acting along the same line as the first force. However, on the top and bottom legs, the forces do not act along the same line, and so there will be some net torque. Remembering that
$\tau = rF \sin\theta_{r,F}$ (RHR),
divining that r is $l/2$ (since the loop will most likely rotate about its centre of mass, although in fact this doesn't really matter) and that $\theta_{r,F}$ is the same as our original θ,
$\tau_{top} = [l/2][IwB] \sin\theta$ (into the page)
$\tau_{bottom} = [l/2][IwB] \sin\theta$ (also into the page),
and the total torque is
$\tau_{total} = lIwB \sin\theta$ (into the page).
If there had been N turns in the loop, the torque would have been N times larger, so let's work that into the relationship. Also, *lw* is the area of the loop, and it can be shown that the result valid for this specific case is valid for any flat loop of area A:
$\tau_{total} = NIAB \sin\theta$ (into the page).
Now, we will do what we did in a similar situation back in Section 1. We will define a new vector, $\boldsymbol{\mu}$, as the *magnetic dipole moment*; its magnitude will be NIA and its direction will be from the centre of the loop toward the 'north pole' formed by the current. We see, then, that the *theta* in our result is the angle between **B** and $\boldsymbol{\mu}$, and so a Right-Hand-Rule seems appropriate. Try **B** x $\boldsymbol{\mu}$; that points out of the page, so let's make it $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$, or,
$\tau = \mu B \sin\theta_{\mu,B}$ (RHR).

Loops which are not flat can of course also have magnetic moments, but they must be calculated a bit more carefully. Here is an example:

Consider a circular loop of radius R which has been bent at a right angle along a diameter. What is the magnetic moment of this current distribution?



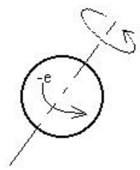
Answer

One last bit of advice: don't confuse the magnetic dipole moment μ with out magnetic constant, μ_o.

Now, let's look at this a different way: we can consider a torque acting to rotate our current loop, thus causing an angular acceleration, or we can develop a potential energy function, and consider said PE to be converted into kinetic energy as the loop swings around. We will make use of the result we obtained for the electric dipole and guess that
$PE_{dipole} = -\mu B \cos\theta_{\mu,B}$.
Careful experimentation confirms that this guess is acceptable.

Causes of Magnetic Effects in Materials

Now, we can return to the beginning of this section and make an attempt to explain the effect which allows some materials to be magnets; this is called *ferromagnetism*, since the largest such effect is seen in iron (Fe). We saw in the last section that current loops can produce magnetic fields which resemble those of bar magnets, and so the loops are affected by other magnetic fields in the same way that magnets are. Consider an electron, a simple model of which is a rotating, charged sphere (with an as yet unmeasurably tiny radius) orbiting the nucleus in much that same way that a planet orbits the sun. The moving charge on the electron is similar to charge moving around a loop, and so a magnetic field should be produced.



Which end of the electron will act like a north pole?

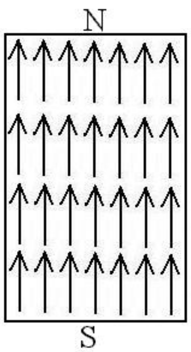
Answer

Let's represent these particles with a little arrow, with the head of the arrow being the north pole and the tail the south pole. Think of it as the electron's magnetic moment. As we progress through the periodic table, electrons fill the orbitals, usually quickly pairing as 'spin-up' and 'spin-down' and thus cancelling any net magnetic moment. Magnetic effects are seen in materials where there is an unpaired electron. This then excludes the noble gases, most even numbered elements, most ions (which have either gained or shed electrons to obtain filled outer shells) and materials which form co-valent bonds (since electrons from two atoms pair up in the bond). The best candidates are elements in the transition columns of the table.

If we were to place two such atoms next to each other, we would expect them to align in opposite directions, since the north poles would attract the south poles, or because this would be the lower energy state:

However, in several materials, the energy considerations are such that a lower energy state is achieved if the spins align parallel to one another:

This occurs in only five elements (iron, nickel, cobalt, gadolinium, and dysprosium), plus a number of alloys. Such materials are said to be *ferromagnetic*. Since this results in an alignment of a majority of these tiny magnets in the same direction, the result is one big magnet:

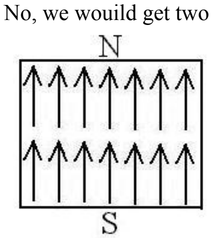


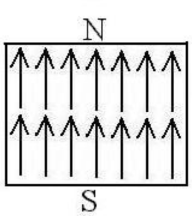
Now, of course, in real life, not all will line up, and there are some fairly easy statistical calculations which can be made for different conditions.

Consider these questions:

What happens if a magnet gets broken? Do we get a (separate) north pole and a south pole?

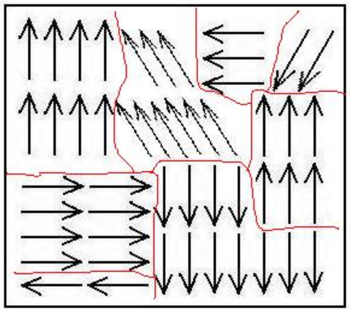
No, we wouild get two magnets:





Why are not all pieces of iron magnets?

The magnetic forces causing this ordering are short-range forces, that is, the ordering is local only. In most iron, there are regions called *magnetic domains* in which a majority of the moments are aligned, but the overall average magnetization is zero (or close to it).



How can we make a magnet?

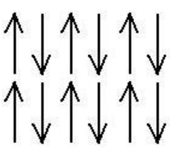
We can place the iron in a strong external field. Then, just like little compass needles, the magnetic moments will align with that field. When the field is removed, the moments are already aligned in such as manner that their interactions reinforce each other, which will preserve the alignment in that direction.

How can we destroy a magnet?

Any process which acts to disrupt the interaction between atoms should do the trick. First, heating the iron introduces thermal energywhen the thermal energy is larger than the associated magnetic energy, the spins will align randomly. The temperature above which ferromagnetism disappears is called the *Curie temperature*. Above the Curie temperature, ferromagnetic materials become *paramagnetic*.

Paramagnetism occurs when there is relatively little or no interaction between adjacent magnetic moments, and so they are arranged randomly. When exposed to an external magnetic field, the moments will align along the field, just as would compass needles, but they will randomize again when the field is removed.

Anti-ferromagnetism is a bit more common than ferromagnetism. In this case, the adjacent moments do align in opposite directions. An example is NiO.



If an external B-field is applied to an anti-ferromagnet, what will happen?

Answer

What will happen when the external field is turned off?

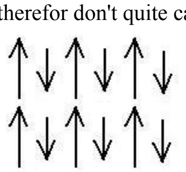
Answer

What will happen if the temperature of the anti-ferromagnet is raised?

Answer

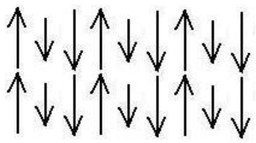
The temperature at which this occurs is called the *Neel temperature*.

In *ferrimagnetism*, there are two (or more) types of magnetic atoms which have different magnetic moments which are anti-parallel, and which therefor don't quite cancel:



This means that these materials can be very magnetic, and indeed, most commercial ceramic magnets (like the ones on your refrigerator) are ferrimagnetic.

In fact, out prototype natural magnet, lodestone (magnetite Fe_3O_4) is a very complicated ferrimagnet, with <u>three</u> species of magnetic atoms, all iron. First, there are doubly ionized iron atoms which occupy what are called 6-f sites in the crystal lattice, and there are trebly ionized irons which occupy two different types of locations in the crystal lattice, 6-f and 4-f sites. A schematic of this system might look like this:



[***Note: the notes on dimagnetism are not complete; in particular, you should ignore the formulas, which still contain errors!]***

Unlike the effects list above, *diamagnetism* is an orbital effect, rather than a spin effect. Here is a simple classical model: Consider two electrons in the same orbital of an atom. We consider them to be orbiting in opposite directions (that is, one has positive and the other negative orbital angular momentum). The magnetic moment of each is calculated from
$\mu = IA = [q/t][\pi r^2] = [qv/d]\pi r^2 = [qv/2\pi r]\pi r^2 = qvr/2$
and the directions of the moments are given by the RHR, and so add to zero.
FIGURE
The centripetal force for each is provided by the coulomb attraction from the positive nucleus:
$k_e qQ/r^2 = mv^2/r$.
Now, let's add a magnetic field B into the page. There will then be a magnetic force acting on each electron (qvB), inward on the clockwise moving electron (1) and outward on the counterclockwise moving electron (2); these additional forces change the centripetal forces, moving (1) into a tighter orbit and (2) electron into a bigger orbit:
$k_e qQ/r_1^2 + qvB = mv^2/r_1$
$k_e qQ/r_2^2 - qvB = mv^2/r_2$
We'll assume that v doesn't change, although that may well not be true.
Continuing along this line requires solving a messy quadratic, but all we need is to see that the two orbit radiuses will be different, $r_1<r_2$. The magnetic moment due to orbital motion will be given by
$\mu = IA = [q/t]\pi r^2 = q[v/2\pi r]\pi r^2 = qvr/2$
So, m1 is
Since the radiuses of the orbits are now different, so are the magnetic moments; the net magnetic moment actually points opposite to the applied magnetic field. As a result, the total magnetic field inside a diamagnetic material is reduced, in a manner similar to what happens to the electric field in a dielectric. In fact, in the same way that conductors act to eliminate any electric fields in their interiors, superconductors are perfectly diamagnetic, so that the total magnetic field in a superconductor is zero.

Note: There are, in fact, two magnetic fields: the magnetic field proper, usually symbolized by **H**, and the *magnetic induction field*, **B**. While related, the two fields represent different properties of magnetism. In this class, we will deal almost exclusively with the induction field, **B**, but like almost everyone else, we will refer to it incorrectly as the magnetic field.

Continue on to the Next Section
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