1-2 Soln

We can do this two ways: time dilation or length contraction. Let d_o be the distance 25 m in the lab, and t_o be the proper lifetime of 2×10^{-8} seconds.

Time dilation: Let t_L be the lifetime of the moving pions as seen from the laboratory point of view. Then,

$$d_o = vt_L = v(\gamma t_o) = (c\beta)\frac{t_o}{\sqrt{1-\beta^2}}$$

$$d_o{}^2 = \frac{c^2\beta^2 t_o^2}{1-\beta^2}$$

$$(1-\beta^2)d_o{}^2 = c^2\beta^2 t_o^2$$

$$d_o{}^2 - d_o{}^2\beta^2 = c^2 t_o^2\beta^2$$

$$d_o{}^2 = c^2 t_o^2\beta^2 + d_o{}^2\beta^2$$

$$d_o{}^2 = \left(c^2 t_o^2 + d_o{}^2\right)\beta^2$$

$$\beta^2 = \frac{d_o^2}{\left(c^2 t_o^2 + d_o{}^2\right)}$$

$$\beta = \sqrt{\frac{d_o^2}{\left(c^2 t_o^2 + d_o{}^2\right)}} = \sqrt{\frac{25^2}{((3\times10^8)^2(2\times10^{-8})^2 + 25^2)}} = 0.946$$

$$v = \beta c = 2.84\times10^8\ \mathrm{m/s}$$

Length contraction: Let d_P be the length of the room as seen by the pion.

$$d_P = vt_o$$

$$\gamma^{-1}d_o = vt_o$$

$$\sqrt{1-\beta^2}\,d_o = c\beta\, t_o$$

$$(1-\beta^2)d_o^2 = c^2\beta^2\, t_o^2$$

$$d_o{}^2 - d_o{}^2\beta^2 = c^2 t_o^2\beta^2$$

$$d_o{}^2 = c^2 t_o^2\beta^2 + d_o{}^2\beta^2$$

$$d_o{}^2 = \left(c^2 t_o^2 + d_o{}^2\right)\beta^2$$

$$\beta^2 = \frac{d_o^2}{\left(c^2 t_o^2 + d_o{}^2\right)}$$

$$\beta = \sqrt{\frac{d_o^2}{\left(c^2 t_o^2 + {d_o}^2\right)}} = \sqrt{\frac{25^2}{((3 \times 10^8)^2 (2 \times 10^{-8})^2 + 25^2)}} = 0.946$$

$$v = \beta c = 2.84 \times 10^8 \text{ m/s}$$