Soln 1-6

Work done equals the change in kinetic energy. Note that v = cβ, so that $K_{Newtonian} = ½m_oc^2β^2$.

a) $W_{Relativistic} = \Delta K = (\gamma_{0.12} - 1)m_oc^2 - (\gamma_{0.10} - 1)m_oc^2 = (\gamma_{0.12} - \gamma_{0.10})m_oc^2 =$

$$\left((1 - 0.12^2)^{-\frac{1}{2}} - (1 - 0.10^2)^{-\frac{1}{2}}\right)m_oc^2 = 0.00224 \times 1 \times (3 \times 10^8)^2$$

$$= 2.096 \times 10^{14}\ \mathrm{J}$$

b) $W_{Newtonian} = \Delta K = ½m_oc^2\beta_{0.12}^2 - ½m_oc^2\beta_{0.10}^2 = (\beta_{0.12}^2 - \beta_{0.10}^2)½m_oc^2 =$

$$(0.12^2 - 0.10^2)½(1)(3 \times 10^8)^2 = 1.980\ 10^{14}\ \mathrm{J}$$

Notice that the difference between the Newtonian and relativistic results is fairly small, even at 10% of the speed of light.

c) $W_{Relativis} = \Delta K = (\gamma_{0.97} - 1)m_oc^2 - (\gamma_{0.95} - 1)m_oc^2 = (\gamma_{0.97} - \gamma_{0.95})m_oc^2 =$

$$\left((1 - 0.97^2)^{-\frac{1}{2}} - (1 - 0.95^2)^{-\frac{1}{2}}\right)m_oc^2 = 0.91089 \times 1 \times (3 \times 10^8)^2$$

$$= 8.198 \times 10^{16}\ \mathrm{J}$$

d) $W_{Newtonian} = \Delta K = ½m_oc^2\beta_{0.97}^2 - ½m_oc^2\beta_{0.95}^2 = (\beta_{0.97}^2 - \beta_{0.95}^2)½m_oc^2 =$

$$(0.97^2 - 0.95^2)½(1)(3 \times 10^8)^2 = 1.728\ 10^{16}\ \mathrm{J}$$

Here, we see quite a large difference.