HW 11-1 Soln)

The normalized wavefunction is

$$\psi_2(x) = \sqrt{\frac{2}{L}}\,\sin\left(\frac{2\pi}{L}x\right)\;;\quad \psi_2^2 = \frac{2}{L}\sin^2\left(\frac{2\pi}{L}x\right)\;.$$

Then,

$$\int_0^{L/3}\psi_2^2\,dx = \int_0^{L/3}\frac{2}{L}\sin^2\left(\frac{2\pi}{L}x\right) = \frac{2}{L}\left[\frac{x}{2} - \frac{\sin\left(\frac{4\pi x}{L}\right)}{\frac{8\pi}{L}}\right]_0^{L/3} = \frac{2}{L}\left(\frac{\frac{L}{3}}{2} - \frac{\sin\left(\frac{4\pi}{3}\right)}{\frac{8\pi}{L}}\right)$$

$$= \frac{1}{3} - \frac{\sin\left(\frac{4\pi}{3}\right)}{4\pi} = 0.402 = \text{40.2 \%.}$$

The first term should be the classical probability.