11-2 HW Soln)

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \tfrac{1}{2}C\,x^2\,\psi = E\psi\,.$$

We're guessing that

$$\psi_1(x) = A_1 x e^{-\beta x^2}.$$

We'll start by taking derivatives:

$$\frac{d\psi_1(x)}{dx} = A_1 e^{-\beta x^2} + A_1 x(-2\beta x)e^{-\beta x^2} = A_1(1 - 2\beta x^2)e^{-\beta x^2}\ ;$$

$$\frac{d^2\psi_1(x)}{dx^2} = A_1(-6\beta x + 4\beta^2 x^3)e^{-\beta x^2} = (-6\beta + 4\beta^2 x^2)A_1 x e^{-\beta x^2} = (-6\beta + 4\beta^2 x^2)\psi_1\,.$$

Substitute in the Sch Eq:

$$\frac{-\hbar^2}{2m}(-6\beta + 4\beta^2 x^2)\psi_1 + \tfrac{1}{2}Cx^2\,\psi_1 = E\psi_1\,.$$

$$\frac{3\,\hbar^2}{m}\beta - \frac{2\,\hbar^2}{m}\beta^2 x^2 + \tfrac{1}{2}Cx^2 = E\,.$$

The constant terms and those quadratic in x must be independently equal:

$$-\frac{2\,\hbar^2}{m}\beta^2 + \tfrac{1}{2}C = 0 \quad ; \quad \frac{3\,\hbar^2}{m}\beta = E\,.$$

From the first equation, we get that

$$\beta = \frac{\sqrt{Cm}}{2\hbar}\ ,$$

and then,

$$E = \frac{3\,\hbar^2}{m}\beta = \frac{3\,\hbar^2}{m}\frac{\sqrt{Cm}}{2\hbar} = \frac{3}{2}\hbar\sqrt{\frac{C}{m}} = \frac{3}{2}\hbar\omega_o\quad .$$

So,

$$\psi_1(x) = A_1 x e^{-\frac{\sqrt{Cm}}{2\hbar}x^2}$$

is a solution if the corresponding energy is

$$E = \frac{3}{2}\hbar\omega_o\quad .$$