HW 13-1 Soln)

The wavefunction for the n = 1, l = 0, m_l = 0 state is

$$\psi_{1,0,0} = A_{1,0,0}R_{1,0}P_0^0e^0 = A_{1,0,0}\, e^{-r/a_o}.$$

Let's substitute into the Schrödinger equation. Any derivative with respect to θ or ϕ will result in zero. That leaves

$$\frac{-\hbar^2}{2m}\left(\frac{2}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{\partial r^2}\right) - \frac{k_eZe^2}{r}\psi = E\psi\,.$$

The first derivative is

$$\frac{\partial\psi}{\partial r} = A_{1,0,0}\frac{-1}{a_o}e^{-r/a_o}$$

and the second is

$$\frac{\partial^2\psi}{\partial r^2} = A_{1,0,0}\left(\frac{-1}{a_o}\right)^2 e^{-r/a_o} = \frac{A_{1,0,0}}{a_o^2}e^{-r/a_o}\quad.$$

Then, we have

$$\frac{-\hbar^2}{2m}\left(\frac{2}{r}\left(A_{1,0,0}\frac{-1}{a_o}e^{-r/a_o}\right) + \left(\frac{A_{1,0,0}}{a_o^2}e^{-r/a_o}\right)\right) - \frac{k_ee^2}{r}\left(A_{1,0,0}\, e^{-r/a_o}\right) = E\left(A_{1,0,0}\, e^{-r/a_o}\right);$$

$$\frac{\hbar^2}{ma_o}\frac{1}{r} - \frac{-\hbar^2}{2m}\frac{1}{a_o^2} - \frac{k_ee^2}{r} = E\ .$$

Now, we have two kinds of terms here, ones that are inversely proportional to r and ones that are constant. We should be able to construct two independent equations.

$$\frac{1}{r}\ \text{terms:}\qquad \frac{\hbar^2}{ma_o} - k_ee^2 = 0 \quad\rightarrow\quad a_o = \frac{\hbar^2}{mk_ee^2}\quad.$$

We recognize this as the Bohr radius, as was asserted in the notes.

$$\text{constant terms:}\quad \frac{-\hbar^2}{2ma_o^2} = E \quad\rightarrow\quad E = -\frac{\hbar^2}{2ma_o^2} = -\frac{(1.055\times10^{-34})^2}{2(9.11\times10^{-31})(5.3\times10^{-11})^2}$$
$$= -2.176\times10^{-18}\ \text{J} = -13.6\ \text{eV}\quad.$$