HW 13-2 Soln)

First, what is the correct value for L_z? In this case, $m_l = 2$ and so $L_z = m_l \hbar = 2\hbar$.

The wave function is

$$\psi_{3,2,2} = A_{3,2,2}\, R_{3,2} P_2^2 e^{2i\varphi} = A_{3,2,2} \left(\frac{r^2}{a_o^2}\right) e^{-r/3a_o} \sin^2\theta\, e^{2i\varphi}$$

The derivative wrt φ is

$$\frac{\partial \psi_{3,2,2}}{\partial \varphi} = \frac{\partial}{\partial \varphi} A_{3,2,2} \left(\frac{r^2}{a_o^2}\right) e^{-r/3a_o} \sin^2\theta\, e^{2i\varphi} = A_{3,2,2} \left(\frac{r^2}{a_o^2}\right) e^{-r/3a_o} \sin^2\theta\, \frac{\partial e^{2i\varphi}}{\partial \varphi}$$
$$= 2i \frac{\partial}{\partial \varphi} A_{3,2,2} \left(\frac{r^2}{a_o^2}\right) e^{-r/3a_o} \sin^2\theta\, e^{2i\varphi} = 2i\psi_{3,2,2} \quad .$$

Then,

$$\widehat{L_z}\, \psi_{3,2,2} = -1\hbar \frac{\partial \psi_{3,2,2}}{\partial \varphi} = -1\hbar 2i \psi_{3,2,2} = 2\hbar\, \psi_{3,2,2} = L_z \psi_{3,2,2}$$

and so,

$$\boxed{L_z = 2\hbar}$$