HW4-7 Soln)

Theisen's distribution is $\rho(f, T) = C_1 T^{1/2} f^{3.5} \exp(-C_2 f/T)$.

To test if the distribution agrees with Stefan's observations, integrate:

$$P = C_1 \int_0^\infty \rho(f, T)\, df = C_1 \int_0^\infty T^{1/2} f^{3.5}\, e^{-C_2 f/T}\, df\ .$$

Let $u = C_2 f/T$ and $du = C_2/T\ df$. Then

$$P = C_1 \int_0^\infty T^{1/2} (uT/C_2)^{3.5}\, e^{-u} \left(\frac{T}{C_2} du\right) = \left[\frac{C_1}{C_2^{4.5}} \int_0^\infty u^{3.5}\, e^{-u}\, du\right] T^5\ .$$

The quantity in brackets is constant, so $P \sim T^5$, in <u>dis</u>agreement with Stefan. FAIL!

To test compatibility with Wien's displacement law, convert the distribution to being in terms of wavelength, differentiate, and set to zero. But first, we're going to change the variable:

$$f = \frac{c}{\lambda} \quad \text{and } df = -\frac{c}{\lambda^2}$$

$$\rho(f, T)\, df = C_1 T^{1/2} f^{3.5}\, e^{-C_2 f/T} df = C_1 T^{1/2} \left(\frac{c}{\lambda}\right)^{3.5} e^{-cC_2/\lambda T} \left(\frac{c}{\lambda^2}\right) d\lambda = \rho(\lambda, T)\, d\lambda$$

$$\rho(\lambda, T) = C_1 T^{1/2} c^{4.5} \lambda^{-5.5} e^{-cC_2/\lambda T}$$

We can ignore the constants in front for the differentiation. Then, set

$$\frac{d\rho}{d\lambda} = \frac{d}{d\lambda}\left(\lambda^{-5.5} e^{-cC_2/\lambda T}\right) = (-5.5)\lambda^{-6.5} e^{-\frac{cC_2}{\lambda T}} + \lambda^{-5.5} e^{-\frac{cC_2}{\lambda T}} \left(+\frac{cC_2}{\lambda^2 T}\right) = 0\ .$$

$$(\lambda^{-7.5})\left(e^{-\frac{cC_2}{\lambda T}}\right)\left(-5.5\,\lambda + \frac{cC_2}{T}\right) = 0$$

One solution is λ = infinity. Another is λ = zero. The third solution is when

$$5.5\,\lambda_{MAX} = \frac{cC_2}{T}$$

$$\lambda_{MAX} = C_3\, T^{-1}$$

This distribution is consistent with the Wein displacement test. PASS!