1-2 Soln

We can do this two ways: time dilation or length contraction. Let do be the distance 25 m in the lab, and to be the proper lifetime of 2×10-8 seconds.

Time dilation: Let tL be the lifetime of the moving pions as seen from the laboratory point of view. Then,

$$d\_{o}=vt\_{L}=v\left(γt\_{o}\right)= \left(cβ\right)\frac{t\_{o}}{\sqrt{1-β^{2}}}$$

$$d\_{o}^{2}=\frac{c^{2}β^{2}t\_{o}^{2}}{1-β^{2}}$$

$$\left(1-β^{2}\right)d\_{o}^{2}=c^{2}β^{2}t\_{o}^{2}$$

$$d\_{o}^{2}-d\_{o}^{2}β^{2}=c^{2}t\_{o}^{2}β^{2}$$

$$d\_{o}^{2}=c^{2}t\_{o}^{2}β^{2}+d\_{o}^{2}β^{2}$$

$$d\_{o}^{2}=\left(c^{2}t\_{o}^{2}+d\_{o}^{2}\right)β^{2}$$

$$β^{2}=\frac{d\_{o}^{2}}{\left(c^{2}t\_{o}^{2}+d\_{o}^{2}\right)}$$

$$β=\sqrt{\frac{d\_{o}^{2}}{\left(c^{2}t\_{o}^{2}+d\_{o}^{2}\right)}}= \sqrt{\frac{25^{2}}{\left(\left(3×10^{8}\right)^{2}\left(2×10^{-8}\right)^{2}+25^{2}\right)}}=0.946$$

$$v= βc=2.84×10^{8} m/s$$

Length contraction: Let dP be the length of the room as seen by the pion.

$$d\_{P}=vt\_{o}$$

$$γ^{-1}d\_{o}=vt\_{o}$$

$$\sqrt{1-β^{2}} d\_{o}=cβ t\_{o}$$

$$\left(1-β^{2}\right)d\_{o}^{2} =c^{2}β^{2} t\_{o}^{2}$$

$$d\_{o}^{2}-d\_{o}^{2}β^{2}=c^{2}t\_{o}^{2}β^{2}$$

$$d\_{o}^{2}=c^{2}t\_{o}^{2}β^{2}+d\_{o}^{2}β^{2}$$

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