Section 6 – Waves and Particles Behaving Badly II

"That's funny….."

Attributed to Alexander Fleming

The Davisson-Germer Accident

By this point, we've seen that light on occasion must act like a particle, and that the photon carries both energy h*f* and momentum h/λ. It's probably not a surprise then that particles sometimes act like waves.

Let's consider a small particle such as an electron, moving at speed v. Making use of the relation above, we might think that λ = h/p = h/mv. This quantity is known as the object's *De Broglie wavelength*, λ_{dB}. If so, then we should be able to observe many of the behaviors of waves exhibited by particles, such as interference. This effect was seen accidently by Davisson and Germer. When De Broglie's notion became known, they repeated the experiment very rigorously.[1] Here, we'll talk about just a few of their results.

Electrons were accelerated through a potential difference, V, toward a nickel crystal,[2] thereby giving them kinetic energy and momentum. The De Broglie wavelength, if such a thing exists, can be calculated this way:

$$\lambda_{dB} = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}.$$

So, what we'll do is calculate the wavelengths of X-rays that would be diffracted by this crystal in the same directions as these electrons were. The geometry is a bit different for this experiment than our previous discussion for X-rays, so we'll have to work through the conditions for constructive interference again. Keep in mind, though, that crystallography is very complicated and three dimensional.

Suppose that the (100) plane is facing the incoming electron beam; clearly, from the figure, the (100) plane is not the diffracting plane, but there well may be some set of planes that will create constructive interference at some angle, φ. Let's assume that set of unknown planes is tilted from the (100) planes by angle α.

So, from the figure, we see a number of things. First, the angle measured experimentally is φ. Second, the spacing of the 'mystery' planes will be reduced from the expected spacing by a factor

[1] Davisson, C. and L. H. Germer, *Phys. Rev.* **30** (1927) p705.

[2] For simplicity, let's assume for now that this was a single crystal.

of sin α: d = D sin α. We suspect that the Bragg condition for constructive interference holds, so 2 d sinθ = mλ. Substituting:

$$2\,D \sin\alpha \cos\theta = m\lambda\,.$$

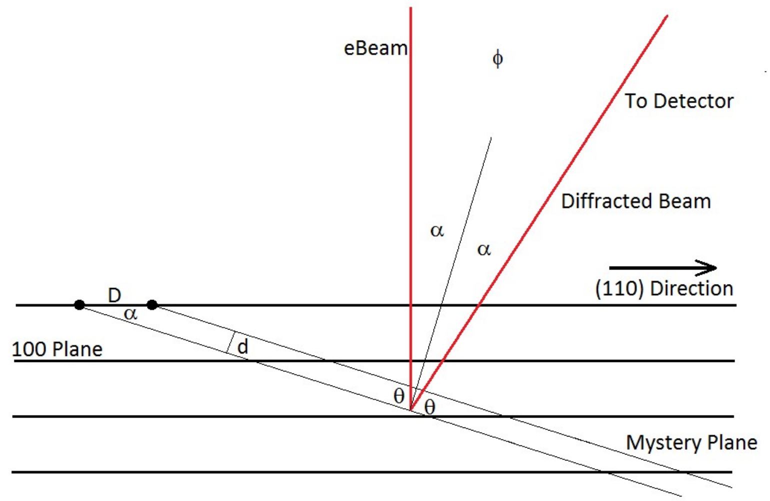
But, alpha and theta are complementary, so we can use a trig identity:

$$2 \sin\alpha \cos\theta = \sin(2\alpha) = \sin\varphi.$$

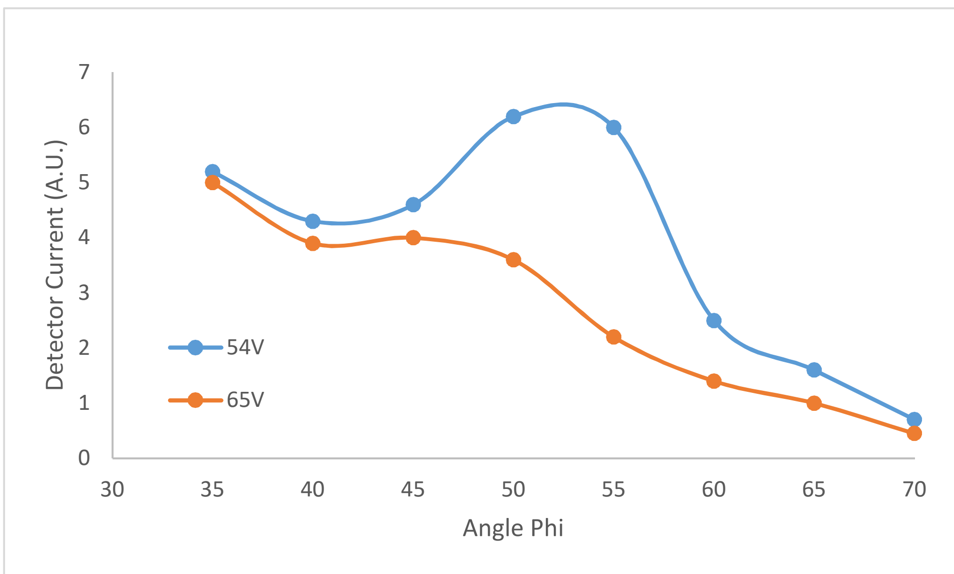
So,

$$D \sin\phi = m\lambda\,,$$

where m is some positive integer.

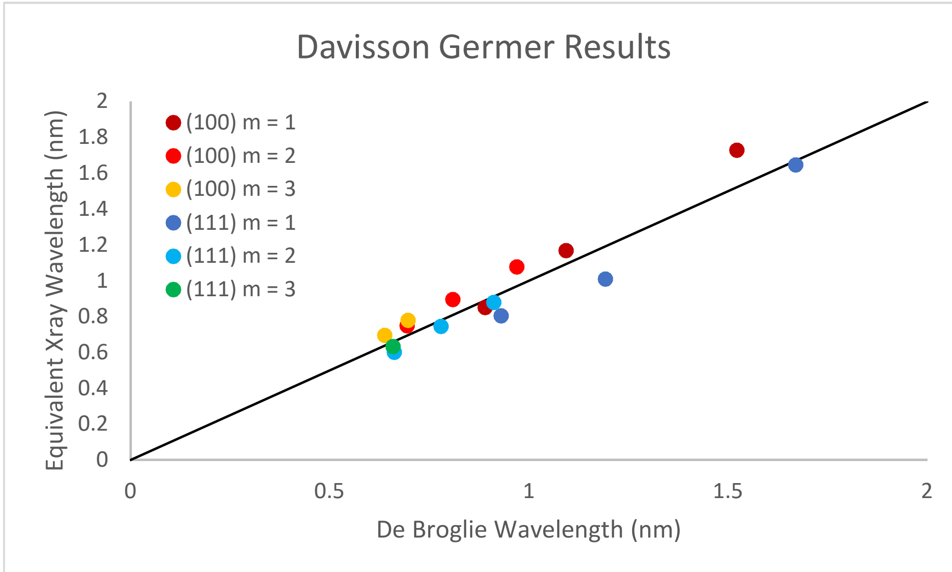


Let's look at some data. First, here is the famous '54 Volt' curve; electrons were accelerated toward the crystal's (111) face by a potential difference of 54 volts. The electron detector was moved in five-degree increments (the curves are simply to guide your eye) and a strong peak is seen at about 50°. For comparison, the 65V curve is also included; the peak is less clear. Accelerating potentials from about 43V to about 66V show some type of peak, however small. By varying the potential and measuring the direction of maximum electron diffraction, we can calculate what wavelength X-rays would have been diffracted in the same direction:



$$\lambda_{\text{XRay Equivalent}} = \frac{\text{D}\sin\phi}{\text{m}}.$$

What we'll do is plot the wavelength of the X-ray that would have been diffracted in a particular direction against the theoretical De Broglie wavelength calculated from the speed of the electrons. The graph (right) shows data for both the (100) and (111) planes. The line has slope one and shows where the two values would be equal. We can see that the two sets of values are fairly equal to one another, demonstrating that the electrons follow the rule for Bragg diffraction, and therefor behave as waves.



Of course, other particles should show the same behavior, but it should become less evident the more massive the object. Shull[3] diffracted 0.0042 eV 'cold' neutrons ($\lambda_{DB} = 4.43$ Å) through single slits of various widths. Since

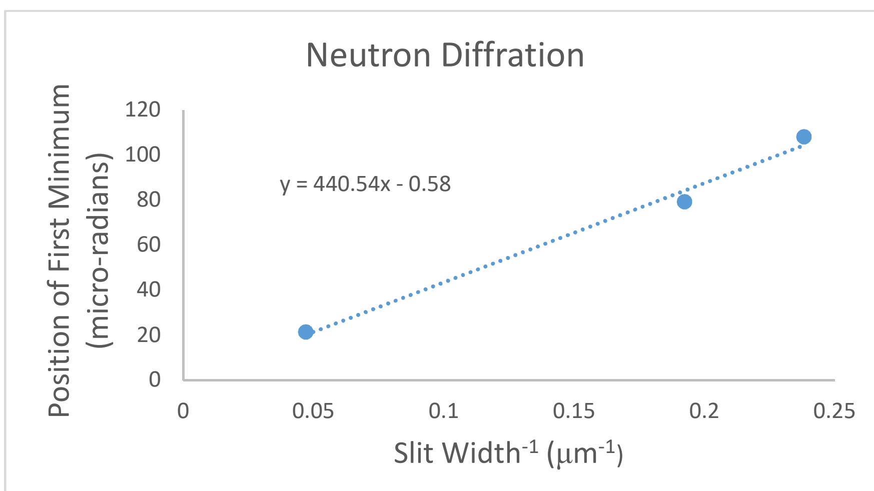
$$b \sin\theta_m = m\lambda_{DB} \quad ,$$

for the minima for light interference, we expect that (at small angles)

$$\theta = m\lambda_{DB}\, b^{-1}$$

Will give us the locations of the

Here are his results for the m=1 minimum. As expected, the intercept is close to zero, and the slope is the wavelength of the neutrons in picometers (to within a half *per cent*).



[3] C.G. Shull, Single slit Diffraction of Neutrons Phys Rev 179 Nr 3 P752 March 1969.

This effect has been seen in particles as large as C_{60}.[4] The authors of this particular study assert that these 'buckyballs' (with a mass over a million times that of an electron) verge on being classical particles.[5]

DISCUSSION 6-1

Why do we not diffract when we walk through a doorway?

HOMEWORK 6-1

A stream of 'slow neutrons' (K = 5 eV) is sent through a narrow slit of width 4μm. What is the angular width (that is, from one side to the other) of the central diffraction maximum?

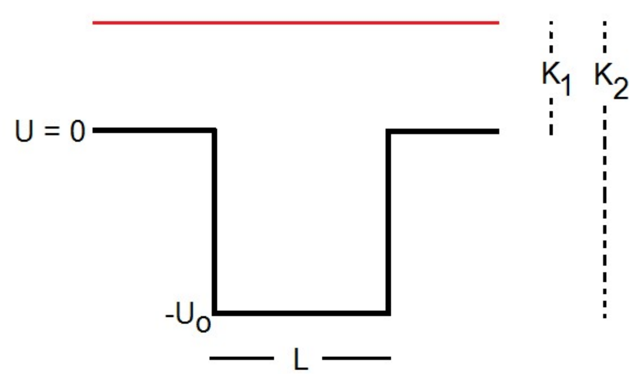
HOMEWORK 6-2

What is the angular width of the central maximum for a bullet going through a door? Take the mass as 5g, the speed as 400 m/s, and the doorway width as 1.2 m.

The Ramsauer Effect

Our last example involves electrons accelerated through a container of gas atoms. If the energy of the electrons is not too great, they will scatter from the atoms elastically. Under certain special conditions, the probability of scattering is minimized. We're going to use an extremely simplified model to predict the conditions for minimal scattering.

Consider xenon, an atom of which is approximately 4 Å in diameter with an electron affinity of -0.8 eV and an ionization energy of 12.13 eV. We'll model this as a one-dimensional problem with the xenon represented as a square potential well. Let's say an electron arrives from the left with kinetic energy K_1. As it arrives at the well, its kinetic energy will increase due to the decrease in potential energy, much like a rollercoaster speeds up as it gets closer to the earth's surface, and the De Broglie wavelength will shorten. Let's find the electron's De Broglie wavelength for each region:



[4] Arndt, Markus, Olaf Nairz, Julian Vos-Andreae, Claudia Keller, Gerbrand van der Zouw, and Anton Zeilinger, 'Wave–particle duality of C_{60} molecules', *Nature* **401** (1999) pp 680-682.
[5] A more recent paper is, Eibenburger, S. *et al*, 'Matter-wave interference with particles selected from a molecular library with masses exceeding 10000 amu,' *Phys. Chem.* **15** (2013) pp 14696-14700.

$$\lambda_{dB\,1} = \frac{h}{p_1} = \frac{h}{\sqrt{2m_e K_1}} \quad \text{and} \quad \lambda_{dB\,2} = \frac{h}{p_2} = \frac{h}{\sqrt{2m_e K_2}} = \frac{h}{\sqrt{2m_e (K_1 + U_o)}} \; .$$

In analogy with light waves traversing a discontinuity, there should be some reflection of the De Broglie wave at each end of the well, with the possibility of a 'phase change.' Since the natures of the two discontinuities are different, we might expect one change to be zero and the other to be a half cycle (equivalent to traveling distance $\lambda_{dB\,2}/2$). Let's consider 'no scattering' to correspond to a weak reflection from a thin film; let the two reflected waves be 180° out of phase. As with light, there is also the consideration that one reflected wave traveled an extra distance 2L. So,

$$\frac{\lambda_{dB\,2}}{2} + 0 + 2L = \left(n + \tfrac{1}{2}\right)\lambda_{dB\,2} \; .$$

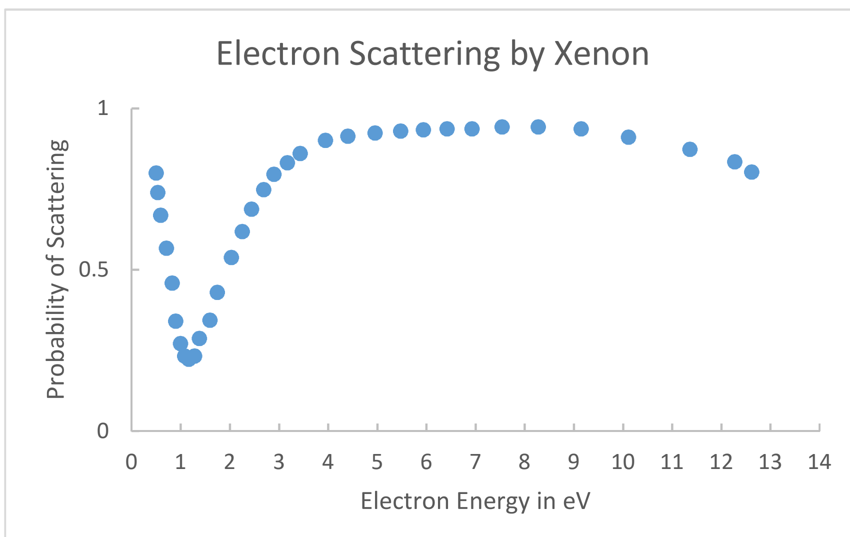
In this equation, n = 0 corresponds to L = 0 (no well), so consider n = 1:

$$2L = \lambda_{dB\,2} \;\; \rightarrow \;\; K_1 = \frac{h^2}{8m_e L^2} - U_o \; .$$

For our example, the well is 4 Å wide and 0.8 eV deep. The kinetic energy of an incoming electron that would experience no reflection should then be, for n = 1,

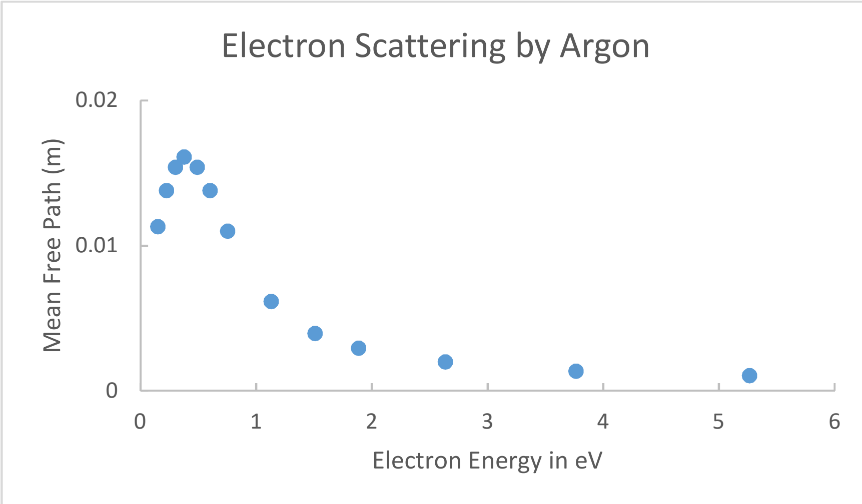
$$K_1 = \frac{(6.63{\times}10^{-34})^2}{8(9.11{\times}10^{-31})(4{\times}10^{-10})^2} \times \left(\frac{1\ \text{eV}}{1.6{\times}10^{-19}\ \text{J}}\right) - 0.8\ \text{eV} = 1.6\ \text{eV} \; .$$

Here are data[6] from an experiment that demonstrate this effect very clearly. The data end at about 12 eV because xenon ionizes. The minimum of scattering occurs for electrons with an energy slightly above 1 eV; considering the crudity of the model used above, this is really not bad agreement. While one may find it conceivable that there may be a second minimum beyond the right edge of the graph, compications too advanced to discuss here preclude it.



HOMEWORK 6-3

[6] Kukolich, S. G., 'Demonstration of the Ramsauer-Townsend Effect in a Xenon Thyratron,' *Am J. Phys.* **36**, 701 (1968).



For argon, the minimum of scattering occurs when the electrons have energy 0.39 eV. The electron affinity of argon is -1.0 Volt. Estimate the diameter of an argon atom.

Note that in this figure, the ordinate is inversely related to the probability of scattering.[7]

The Wrap

We've seen several examples of waves behaving as particles and of particles behaving as waves. So, are these things we've discussed particles or waves? When you figure it out, let me know.

[7] J.S. Townsend and V.A. Bailey, The motion of Electrons in Argon and in Hydrogen, Phil Mag S6 Vol 44 Nr 263 November 1922 P 1033.