Section 7 - Really, how big is the nucleus, and why does that not seem right?

In previous discussions, we found upper limits to the 'radius' of the nucleus. For example, the Rutherford data for the gold nucleus showed that the radius is less than 1.5×10^{-14} m. A better value is about half that. What this really told us is that that is the distance at which the Coulomb force stopped being the only force acting on the alpha particles.

So, let's look at some more data, in this case, electron scattering from the carbon nucleus.[1] You may remember from optics that light passing through a small circular aperture of diameter A results in a diffraction pattern. If instead, we replace the barrier and opening with a circular barrier disk of diameter A, the pattern remains the same. You may remember that the first minimum ring occurs when

$$\theta_1 \approx \frac{1.22\,\lambda}{A}.$$

As we'll see in a moment, the angle is quite large, so we'll have to drop the approximation for small angles to re-obtain

$$\sin\theta_1 = \frac{1.22\,\lambda}{A}.$$

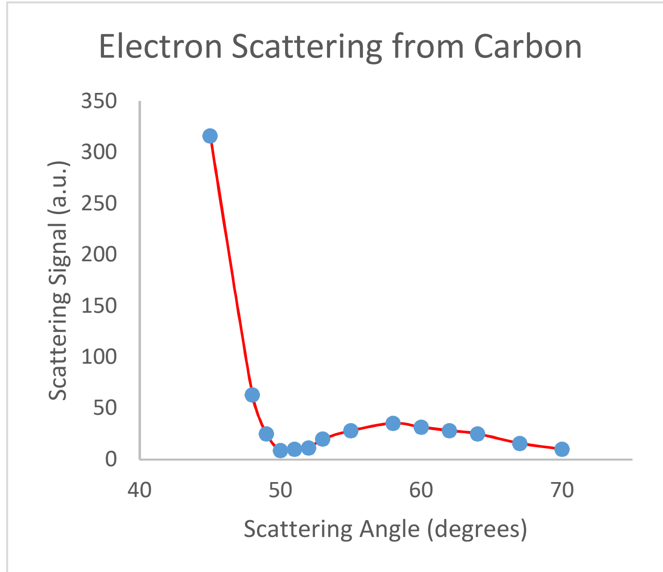
Now, consider the spherical cross-section of the nucleus to be that disk with electrons of a particular De Broglie wavelength incident on it. The scattered electrons should form a pattern on a screen from which we can deduce the diameter of the nucleus. The radius of the nucleus will be given by

$$R = \frac{A}{2} = \frac{1.22\,\lambda_{dB}}{2\sin\theta_1}.$$

EXAMPLE 7-1

[1] Hofstadter, Douglas, 'The electron-scattering method and its application to the structure of nuclei and nucleons,' Nobel Lecture (1961).

Consider the data in the graph. We can see that the minimum occurs at a fairly large angle of 51°. Find the radius of the carbon nucleus.



The electrons used in this experiment had kinetic energy 420 MeV, which makes them relativistic. Then,

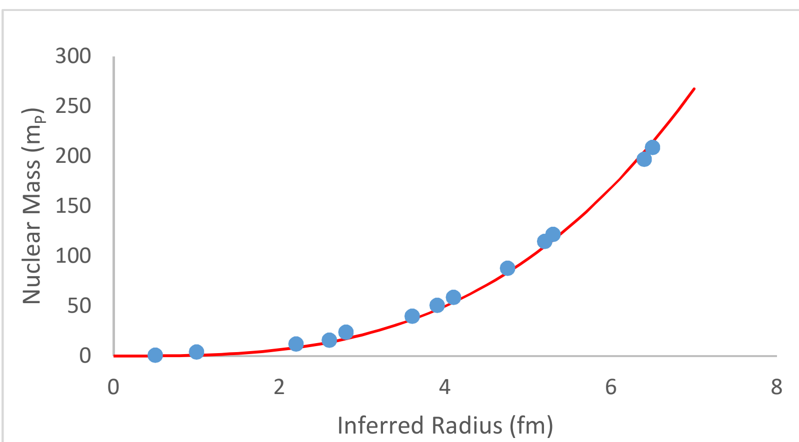
$$K \gg m_e c^2 \quad \rightarrow \quad p \approx \frac{E}{c} = \frac{K}{c}$$

and

$$p = \frac{h}{\lambda_{dB}} \quad \rightarrow \quad \lambda_{dB} = \frac{hc}{K}.$$

So,

$$R = \frac{1.22\,hc}{2\sin\theta_1 K} = \frac{1.22(6.63\times10^{-34})(3\times10^{8})}{2(\sin 51^{o})(420\times10^{6}\times1.6\times10^{-1}\ )} = 2.35\times10^{-15}\text{m}\,.$$ [2]



This experiment was repeated for a number of nuclei, and the respective radiuses were inferred through some modelling. The graph shows that the mass is proportional to the cube of the radius, as may be expected if the densities of all nuclei are the same. The red curve is the fit

$$M = 0.58\,R^3$$

with M in multiples of the proton mass and R in femtometers (10^{-15} m).

HOMEWORK 7-1

Pick a point on the curve and calculate the density of a typical nucleus. What volume of this material would have a mass of one tonne?

[2] Having said all this, there is a correction factor of about 7/6 necessary to account for the observations that the nucleus does not actually have a well-defined surface. So, $R_C \approx 2.75\times10^{-15}$ m.

HOMEWORK 7-2

An experiment similar to the one above was conducted with uranium using 100 MeV electrons. The first diffraction minimum was found at 57°. Calculate the radius of a uranium nucleus. Do <u>not</u> use this value for Homework 7-3.

HOMEWORK 7-3

Use the formula above ($M = 0.58\ R^3$) to find the ratio of the volumes of a C-12 nucleus and of a U-238 nucleus; assume that they are spheres. Next, calculate the ratio of their volumes based on their charges (+6e and +92e, respectively). Why is there a discrepancy?