Section 2-8 - Geometric Optics

The Speed of Light

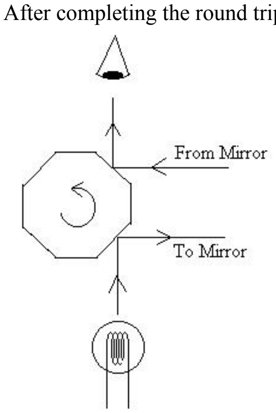
We all know from experience that light travels more quickly than does sound. Consider a flash of lightning, which is then followed some time later by the rumble of the thunder, which was created at the same moment (We know this by observing that near strikes have less of a delay than do more distant strikes). Indeed, we can estimate the distance to a strike in km by counting the number of seconds between flash and clap and dividing by three.

Aristotle concluded that light travels infinitely fast, based on the observation that, when one turns on a lamp, the reflection in a mirror appears immediately. Some describe this as there being no 'echo,' no delay between the light which took a direct path to our eyes and that which was reflected from the mirror.

The first scientific attempt to measure the speed of light was performed by Galileo, who took a lamp to the top of a hill, while his assistant did the same on a different hill. Galileo uncovered his lantern, the assistant uncovered his when he saw Galileo's light, and Galileo measured the time from when he uncovered to when he saw his assistant's light. This experimental technique is still used today and is called a *time-of-flight measurement*. Unfortunately, Galileo was only measuring human reaction times.

The next advance was made by Roemer, who observed slight discrepancies in the time between the eclipses of Jupiter's moons. He concluded that the increase in the period was due to the earth moving in its orbit away from Jupiter, and decreases in the period were due to the earth approaching Jupiter, such that the extra time was that necessary for the light to cross the orbit of the earth, and that the speed of light was therefor not infinite. Huygens used these data and arrived at a value of about 2.3×10^8 m/s (the size of the earth's orbit was not well known, then).

In 1849, Fizeau used a toothed wheel, and later Michelson used a rotating mirror apparatus, to determine that the speed of light was about 3×10^8 m/s. The idea is that light from a bulb is reflected to a mirror some thirty-five km away when the mirror is in the position shown in the figure. After completing the round trip, the mirror must have rotated 45° so that a different face reflects the light to the observer's eye.



If the light is seen for <u>any</u> speed of rotation, then the speed of light would be what?

Answer

Knowing the slowest frequency which allows the light to be viewed, and the optical path length, allows one to calculate the speed of light.

The Nature of Light

Scientists have contemplated the nature of light for many centuries; some of the greatest minds in western civilization have worked on the problem. By the seventeenth century, though, two competing and apparently mutually exclusive theories vied for acceptance. As usual, politics and personalilty played no small part in determining which view scientists of the time adhered to. Hooke and Huygens supported the *wave nature of light*, which successfully predicted the *reflection* and *refraction* of light. Newton was a proponent of Aristotle's *corpuscular theory*, in which light was composed of minute particles, which he claimed also explained reflection and refraction. Newton's interpretation was ascendant for well over a century. As evidence from observation and experimentation accumulated, each side of the controversy tried to fit the new data into the frameworks of the respective theories.

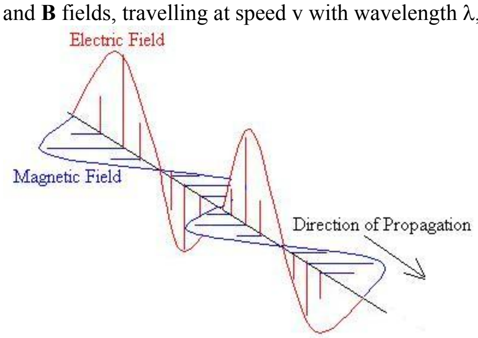
However, by the turn of the ninteenth century, the wave view of the nature of light regained support when Young explained *interference* and *diffraction* effects in terms of waves; these effects simply can not be explained in terms of a particle model. By mid-century, Foucault had shown that Newton's explanation of refraction was based on false assumptions, and Maxwell (1860s) combined Gauss's Law, Ampere's Law, and Faraday's Law to predict the existence of *electro-magnetic waves* with an expected velocity equal to the known speed of light; the implication seems clear: light is an electromagnetic wave.

In the 1880s, Hertz used a *spark-gap* to produce electro-magnetic waves of a type we now call *radio waves*; he demonstrated that such waves exhibit all of the behaviours of light, such as reflection, refraction, *polarization*, and diffraction, and so he surmised that
1) radio waves are EM waves,
2) light behaves in the same manner as radio waves,
3) therefore, light is an EM wave.
Interestingly, Hertz's experiment, which confirmed the wave nature of light, also exhibited different results when conducted with or without the room lights on, a behaviour which could not be explained with the wave model and which we now call the *photo-electric effect*. Einstein was able to explain this effect, but by using a particle-like model for light; these particles are called *photons*. It was left to the more mature outlook of the twentieth century to reconcile these two models; we say now that light is neither wave nor particle, but that it exhibits some behaviours of each.

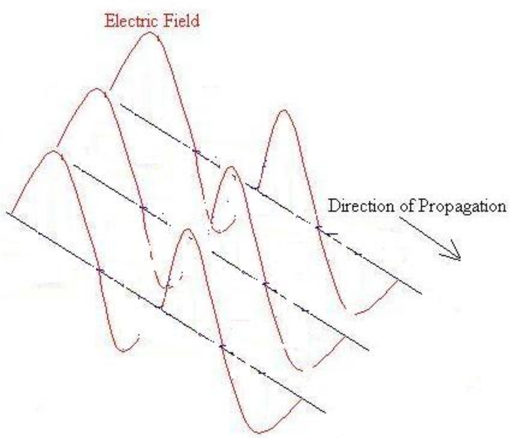
For now, let us consider light to be a wave.

Waves, Wavefronts, and Rays

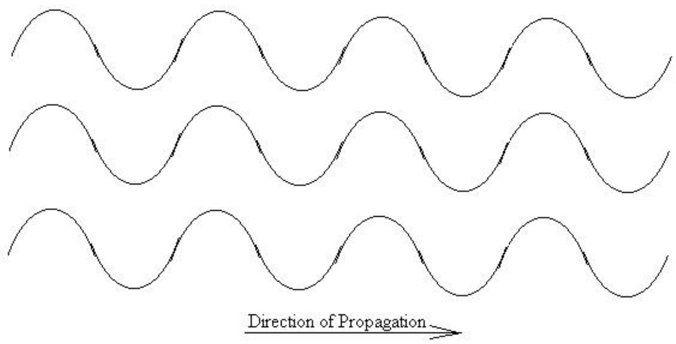
In the last section, we decided for the meantime to accept that light behaves as a wave. Now, how are we to represent the light? The manner in which we do this depends on how much information we need and how much we can ignore. For example, the actual wave is a combination of **E** and **B** fields, travelling at speed v with wavelength λ, with the **E** and **B** fields in a particular orientation, *et c*.



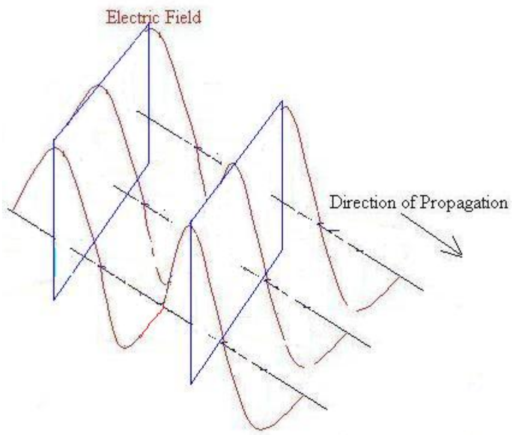
Now, we can eliminate some of this clutter by realizing that the **E** and **B** fields are always at right angles to one another such that **E** x **B** gives the direction of propagation of the wave, that they are always in phase with one another, and that the magnitude of the electric field E at some spot always equals cB (these characteristics are apparent from Maxwell's equations). So, we really only need keep track of what the electric field is doing to preserve all the information about the wave.



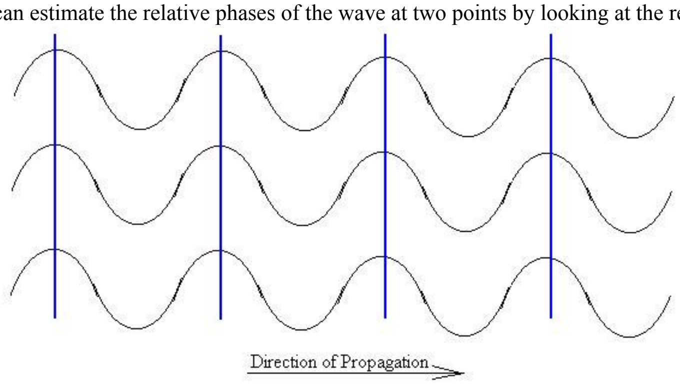
or, more schematically,



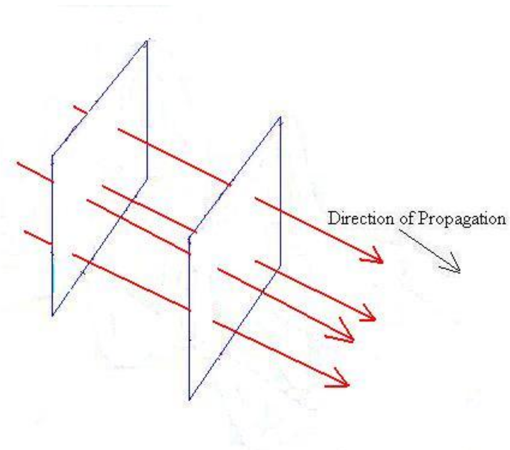
Now, suppose that we don't really care about anything but the direction of propagation, the wavelength, and perhaps the phases of different portions of the wave. We can construct *wavefronts*, surfaces which connect all points with the same phase on adjacent waves (blue surfaces). For example, we could connect all adjacent crests.

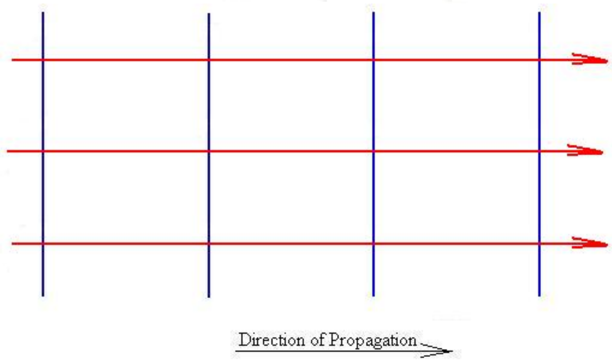


The spacing between wavefronts will then be the wavelength of the wave, the direction of propagation is perpendicular to the wavefronts, and we can estimate the relative phases of the wave at two points by looking at the respective distances from a wavefront surface.



We can simplify even further by dropping the phase information completely and concentrating only on the direction of the light, tracing out the path it takes through space. Such a line is called a *ray*, and it is perhaps the most natural representation; recall seeing the sunlight streaming from a small break in the clouds after a storm. Rays must be at right angles to the wavefronts.

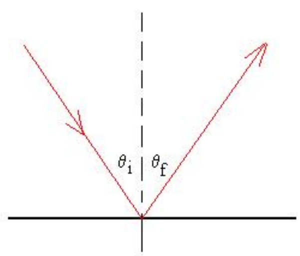




Reflection, Refraction, and a Surprise

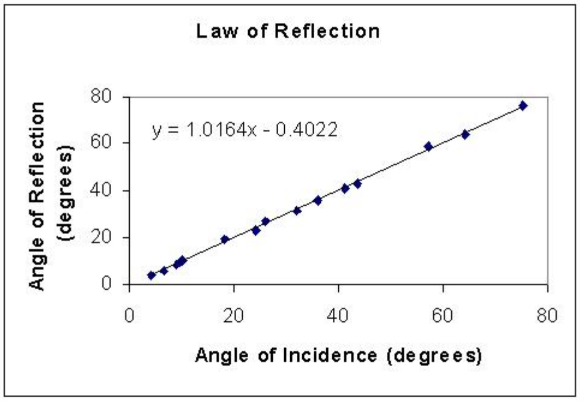
Reflection

If we shine a ray of light on a flat surface, it will be reflected in a manner such that the angle of reflection equals the angle of incidence, $\theta_r = \theta_i$:

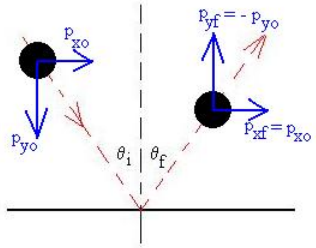


Note that the angles are measured from an imaginary line perpendicular to the surface, a *normal* line.

Here are some experimental results comparing the angles of incidence and reflection (2001/2 class):

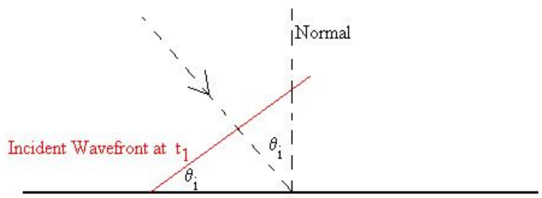


This property could be explained with either the corpuscular or wave theories of light. A ball hitting a wall in an elastic collision and without spin will rebound at an angle equal to the incident angle because of conservation of momentum.

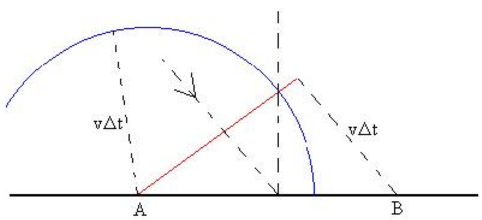


The momentum in the x direction will remain unchanged, and the momentum in the y-direction will completely reversed, and so $\theta_r = \theta_i$.

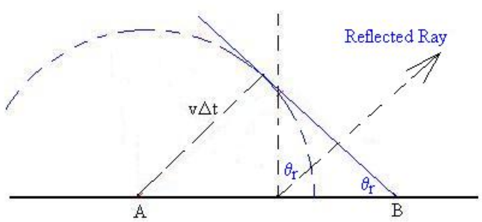
For waves, we first need to discuss *Huygen's Principle*, which states that every point on a wave front can be considered to be a point source for new waves. Then, we consider a ray (with its corresponding perpendicular wavefront) incident at speed v on a flat surface at angle θ_1 as shown. The edge of the wavefront has just reached Point Aat time t_1:



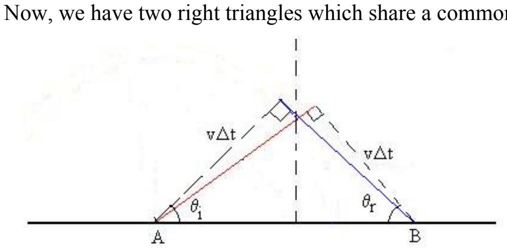
According to Huygen's Principle, Point A will be the source for new waves (Of course, so will many other points on the surface as well, but we'll consider only what happens at Points A & B). Consider the same wavefront a time Δt later, during which time the wave has moved such that the <u>other</u> edge of the wavefront has arrived at Point B. The distance that edge has moved in the interval is vΔt. Now according to Huygen's Principle, Point B is a source for new waves. The wave created at B has not yet had any chance to go anywhere, but the wave from Point A (shown in blue) could be anywhere on a circle of radius vΔt centred on Point A.



The reflected wavefront has to meet two conditions: first, it must be tangent to the circle we drew around A, since it must be perpendicular to its direction of motion away from A, and it must pass through B. We can then also generate a ray indicating the direction of the reflected light. Through some geometry, we knw that the angle at Point B is equal to the angle of reflection, θ_r.



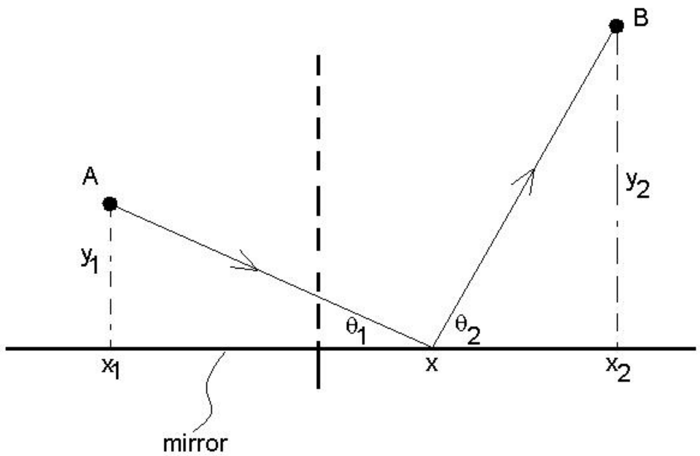
Now, we have two right triangles which share a common hypotenuse (between Points A & B) and which each have a side of length vΔt.



These triangles are therefor congruent, and the angles at Points A and B are equal. So, too, then are the angles of incident and reflection.

Another point of view is *Fermat's Principle of Least Time,* which states that light will take the path from one point to another which requires the least time (Strictly speaking, this is a special case of a principle used in mechanics, where the actual statement is that the time is at an extreme.).

Consider two points, A and B. Light travels from Point A to Point B after reflecting off a mirror along the x axis. Let x represent the spot at which the reflection occurs.



The shortest time in this case also means the shortest distance. For given reflection point x, the distance travelled by the light will be:
$D = [y_1^2 + (x_1 - x)^2]^{1/2} + [y_2^2 + (x_2 - x)^2]^{1/2}$

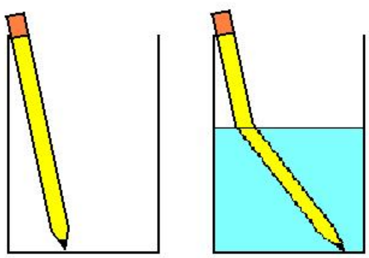
Then set $dD/dx = 0$

$^1/_2[y_1^2 + (x_1 - x)^2]^{-1/2}[2(x_1 - x)(-1)] + {}^1/_2[y_2^2 + (x_2 - x)^2]^{-1/2}[2(x_2 - x)(-1)] = 0$
$- [y_1^2 + (x_1 - x)^2]^{-1/2}(x_1 - x) = [y_2^2 + (x_2 - x)^2]^{-1/2}(x_2 - x) = 0$
$\cos\theta_1 = \cos\theta_2$
$\theta_1 = \theta_2$

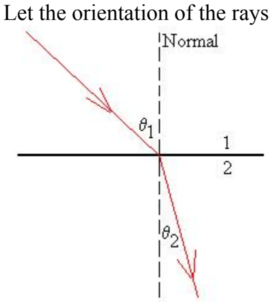
We should of course go back and take the second derivative to show that this corresponds to a minimum.

Refraction

Refraction is another process by which the path of light is bent. You may have noticed that objects under water appear to be closer to the surface than they really are. For example, a pencil partially immersed in water apppears not to be straight (the figure is not quite right; better just to try this at home):

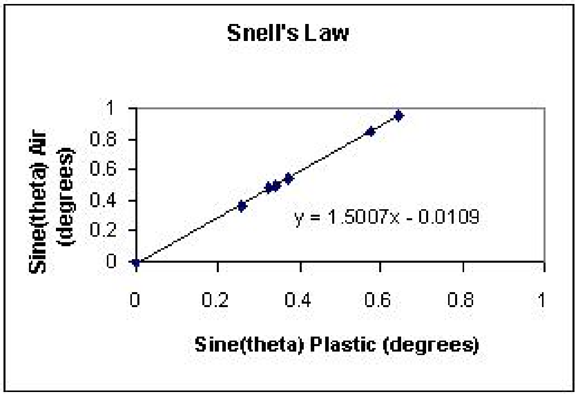


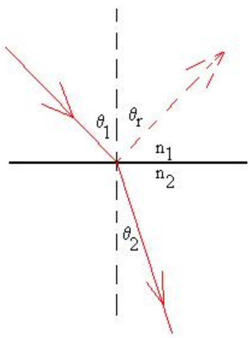
Let the orientation of the rays be measured from the normal to the interface between materials 1 and 2, as described in the reflection section.



Snell found that the ratio $R = \sin\theta_1/\sin\theta_2$ is a constant for any given pair of materials. Interestingly, there is a relationship among the ratios of combinations. Suppose one measures the ratio for three combinations: air to water, water to glass, and air to glass. We find that
$R_{air,water}*R_{water,glass} = R_{air,glass}$.
This leads to the notion that there is a particular value associated with each type of material which is independent of the properties of the other material; this is known as the *index of refraction*, n. For air (well, vacuum, actually), n is set to 1; values for other materials can be determined experimetally. If so, we can re-write our relationship as
$n_1 \sin\theta_1 = n_2 \sin\theta_2$,
which is known as *Snell's Law for Refraction*.

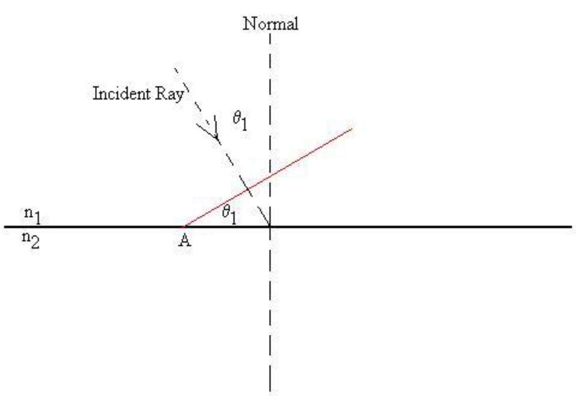
Here are some experimental results comparing the sines of θ_1 and θ_2 for an air-plexiglas interface (class of 2001/2). The line's slope represents $n_{Plexiglas}$:



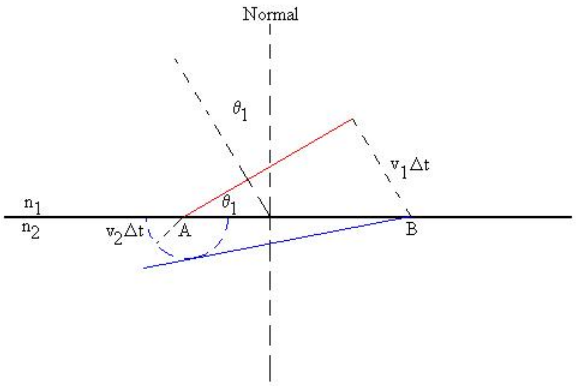


We now know that this effect is caused by the fact that light has different speeds in different materials, or in different parts of the same material (see derivation below). Remember (from above) the wave equation, where we saw that the speed of light (in vacuum) was given by
$c = 1/[\varepsilon_o\mu_o]^{1/2}$.
Think back to the effect of the insertion of a dielectric of constant κ into a capacitor:
$C_{diel} = \kappa\varepsilon_o A/d$.
So, if the light wave were to travel through a material with dielectric constant κ, we would expect to have to modify our velocity result accordingly:
$v = 1/[\kappa\varepsilon_o\mu_o]^{1/2} = c/\kappa^{1/2}$,
that is, the light slows by a factor $n = \kappa^{1/2}$, so that
$v = c/n$;

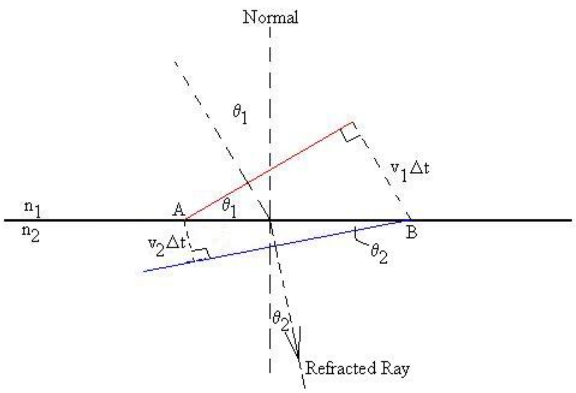
Let's see if we can prove that a change in speed will account for refraction. Consider a wavefront incident on a flat interface at angle θ_1 (red line), for which we can consider the wave at Point A to be the source for new waves.



A time Δt later, the waves created at A are somewhere on the dotted curve a distance $v_2\Delta t$ from A, while the upper end of the wavefront still in material one has traveled a distance $v_1\Delta t$ to reach the interface at Point B. To construct the new wavefront in material two, we know that it must be tangent to the dotted curve (since it must be perpendicular to its own direction of motion) and pass through B (blue line).



We observe the two right triangles which share a hypoteneuse (H) between Points A & B



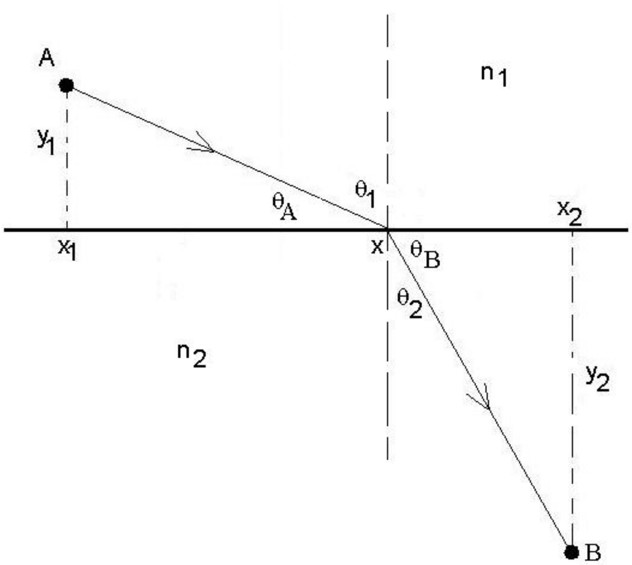
and determine that
$H = v_1\Delta t/\sin\theta_1 = v_2\Delta t/\sin\theta_2$,
$(c/n_1)/\sin\theta_1 = (c/n_2)/\sin\theta_2$,
$n_1\sin\theta_1 = n_2\sin\theta_2$.

Newton gave a detailed explanation of how refraction could be accounted for in the corpuscular theory, but we shall not discuss it here, since Foucault showed it to be based on false assumptions. Realize, though, that it was a well respected theory for over 150 years.

Question:
What would happen if $n_1 = n_2$?

Answer

Let's try this again using Fermat's Theorem.



The time T to travel from A to B will be given by
$T = d_1/v_1 + d_2/v_2 = (n_1/c)[y_1^2 + (x_1 - x)^2]^{1/2} + (n_2/c)[y_2^2 + (x_2 - x)^2]^{1/2}$

Then set $dT/dx = 0$

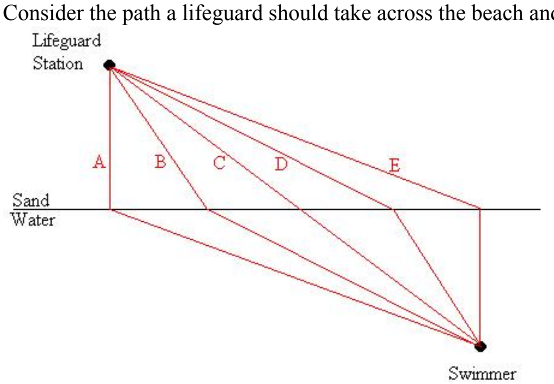
$^1/_2(n_1/c)[y_1^2 + (x_1 - x)^2]^{-1/2}[2(x_1 - x)(-1)] + {}^1/_2(n_2/c)[y_2^2 + (x_2 - x)^2]^{-1/2}[2(x_2 - x)(-1)] = 0$
$- n_1[y_1^2 + (x_1 - x)^2]^{-1/2}(x_1 - x) = n_2[y_2^2 + (x_2 - x)^2]^{-1/2}(x_2 - x) = 0$
$n_1 \cos\theta_A = n_2 \cos\theta_B$
But, θ_A and θ_1 are complementary, as are θ_B and θ_2 , so

$n_1 \sin\theta_1 = n_2\, \sigma\iota\nu\theta_2$

Yee-hah!

Without proof, we assert that
$Z = [\mu_o/\kappa\varepsilon_o]^{1/2} = Z_{vacuum}/n$.
We'll make use of this in Section 9.

Example:
Consider the path a lifeguard should take across the beach and through the water to reach a drowning swimmer as quickly as possible.



Answer

There was a London stage mystery play back in the thirties (of which I'm afraid I have forgotton the title) in which a major plot device was the fact that the stolen jewels had been hidden in a pitcher of water, thus making them invisible. Would this really work?

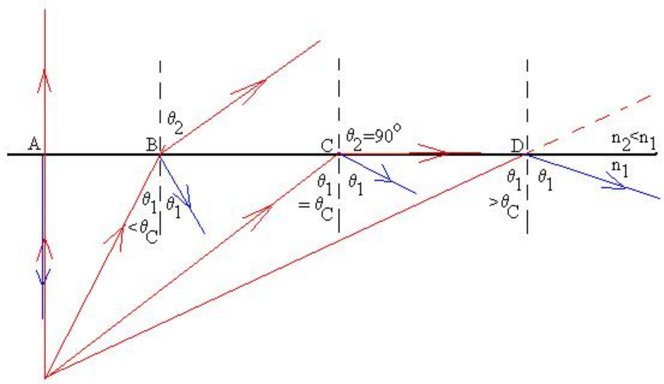
Answer

On the other hand, if two materials have the same indices of refraction, then this trick would work. An example you can try at home is Kimax(Tm) glass and Caro(Tm) corn oil. The indices are fairly close so that it becomes difficult (if not impossible) to see the glass in the oil. To complete the trick, warm the oil slightly until the glass completely disappears. The indices of refraction of materials are generally temperature dependent.

The index of refraction is also generally wavelength dependent. For example, n_{glass} for light with wavelength 400nm is generally higher than for light of wavelength 700nm. As a result, these two wavelengths will be refracted along slightly different paths. If white light comprising all colours is incident on glass, the colours will be spatially separated, forming a *spectrum*. Newton coined this expression, because the light so produced appeared 'ghostly.' Natural examples of such spectra include rainbows and sun-dogs.

Total Internal Reflection

Let's consider a special case of Snell's Law. Suppose that light comes from a material of high index of refraction (in this diagram, the lower material) into a material of lower index (here, on top).

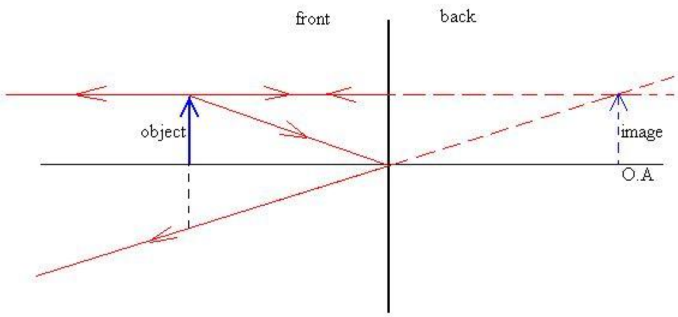


There will be a reflected ray (in blue), and a transmitted (refracted) ray. Since $n_2<n_1$, θ_2 will always be greater than θ_1, and so as we increase θ_1, it will eventually reach a value (called the *critical angle* θ_C) such that $\theta_2 = 90^o$. At that point, no light will be exiting into material two; it will all be reflected back into material one. This effect is referred to as *total internal reflection*. It may seem strange that a wave would travel along the interface, but don't worry, it doesn't; as θ_2 approaches 90^o, the transmitted ray becomes dimmer and dimmer and fades out completely when $\theta_2 = 90^o$.

This effect is used in binoculars and in fibre optic cables. The effect is also responsible for the *brilliance* of cut gemstones , such as diamonds. Light entering the gem is internally reflected many times before exiting; the colour dispersion and multitude of reflecting angles makes the stone appear to sparkle.

Ray Tracing and Mirrors - Plane, Concave, Convex

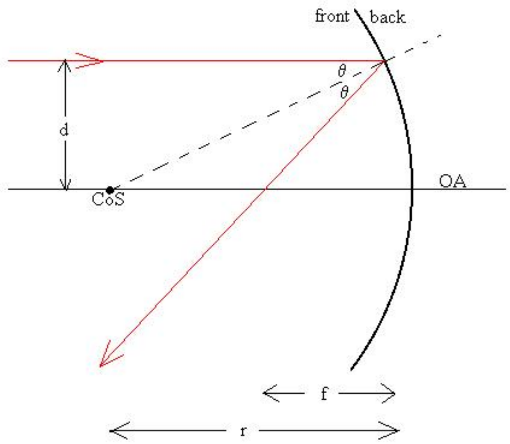
We talked a little about *plane* or *flat mirrors*. What does such an optical element do to light? Let's place an object, represented by an arrow, in front of the mirror.



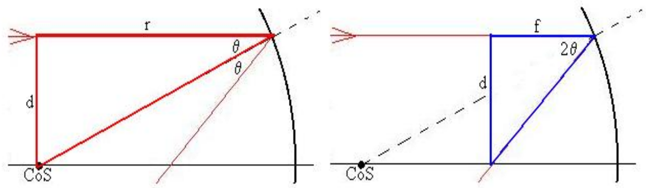
The front and back of the mirror are labeled, and the *optical axis* (OA) is a reference line which runs through the centre of the mirror and perpendicular to its surface. We'll use a method called *ray tracing* to follow the paths of several rays which are easy to calculate: in this case, the ray which leaves the tip of the arrow, hits the mirror at normal incidence (that is, $\theta = 0^o$) and returns along its original path, and the one which hits the mirror at its centre and is reflected at an equal angle, as shown. The latter is chosen because it will pass by the bottom of the arrow at the same distance from teh optical axis that the head of the arrow is above the optical axis, and so it's very easy to draw in. An observer standing over to the left will see these two rays, trace them backwards, and think that they originate from a point behind the mirror. This is where the *image* of this object forms. A little bit of geometry will show that the image forms the same distance behind the mirror that the object is in front of it. Prove this to yourself.

What about mirrors with a curved surface? We shall restrict ourselves to mirror surfaces which are sections of a sphere, since that will make our derivations a little easier, but you should realize that, in real life, the surfaces could be *parabolic* or *hyperbolic* (both common in telescopes) or one of a number of other shapes.

Consider a *concave mirror*, as shown. We shall draw in the optical axis as a reference line; it passes through the centre of the mirror, perdendicular to its plane. If the surface is part of a sphere, there must be a centre to the sphere (CoS), and the distance of this point to the mirror is the *radius of curvature* of the mirror surface, r. Now, let's consider a ray which comes into the mirror parallel to the OA, a small distance d away from it (the derivation we're about to do requires small angles). The ray will hit the mirror surface at an angle θ and be reflected at the same angle from the normal. How do we generate the normal? It will connect that point on the mirror where the reflection takes place to the CoS, *i.e.*, it's a radius of the sphere. The ray will return to the left and eventually cross the OA at a distance we'll call *f* from the mirror.

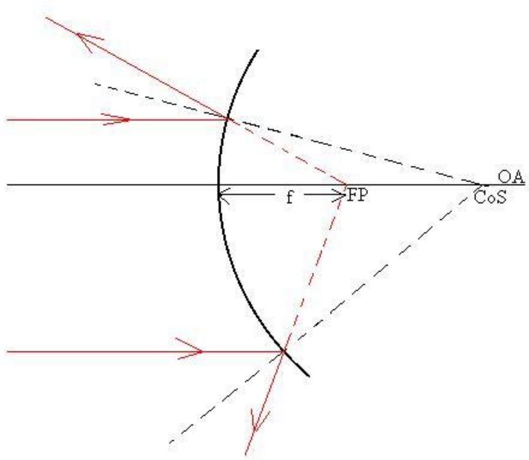


Consider two triangles in this diagram:



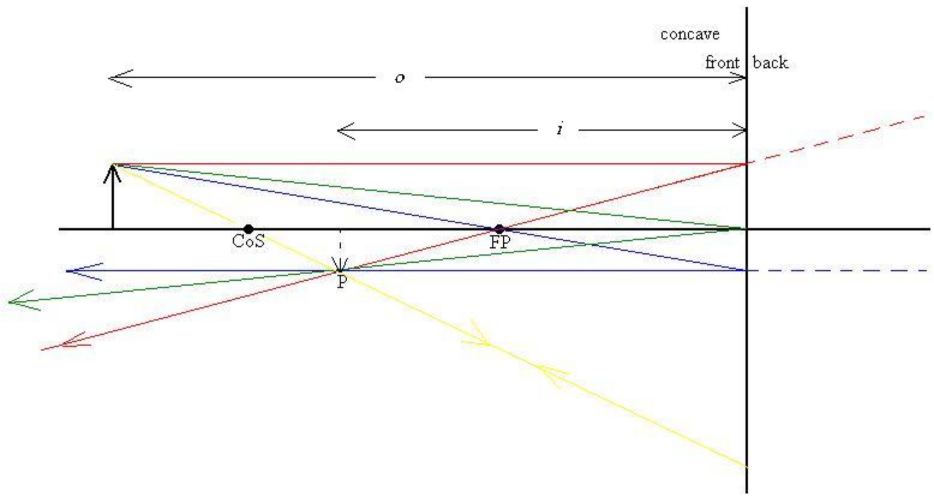
Because of the curvature of the mirror, the distances labelled 'r' and '*f*' are not quite r and *f*, but for small values of d, they will be close.
For the red triangle, we can write that (at least approximately)
$d/r = \tan\theta$.
For the blue triangle, we can write that (at least approximately)
$d/f = \tan(2\theta)$,
but since the angles are small, we can re-write this as
$d/f = 2\tan(\theta)$.
Comparison of the two relations tells us that $f = r/2$, and that this is true regardless of d. This last bit of information tells us that any ray coming in parallel to the OA will be reflected back through this special point, which we shall now call the *focal point*. The distance *f* is then known as the *focal length* of the mirror. Note that this result required a number of approximations, for example, the angles we used must be small, and the tangents of θ and 2θ are actually a little bigger than we said.
In real life, of course, this is not quite true, and *spherical abberation* is the result. Parabolic surfaces do not exhibit such abberation.

Convex mirrors should also have a focal point, although it is behind the surface of the mirror, and the rays don't actually cross there, they only seem to have crossed there:



A similar argument can be used to assert once again that $f = r/2$.
Prove this to yourself.

Let's see if we can use some of this information to predict where an image of an object will form. We shall use an arrow as our object, because it possesses attributes in which we are interested: location, orientation, size, and as we'll see later, reality or non-reality. The technique we're about to use is called *ray tracing*, and it should be done with pencil, paper, and a ruler.
Consider the following situation involving a concave mirror; the mirror is drawn flat to aid in the raytracing process, but keep in mind that the surface is actually curved:



The distance *o* is called the *object distance* and it is measured from the mirror surface. Consider four rays of light which emanate from the head of the arrow:

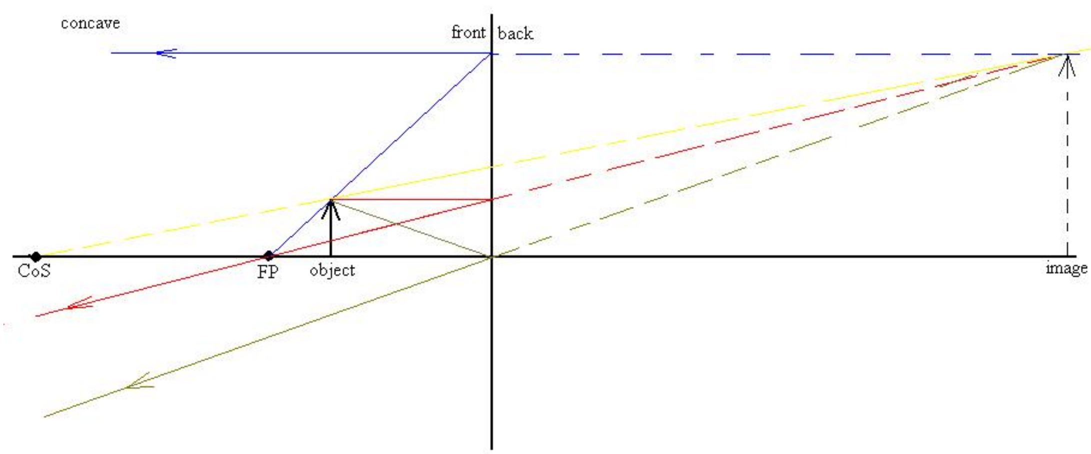
- The red ray leaves the arrow head, comes in parallel to the OA, then is reflected back through the focal point.
- The blue ray leaves the arrow head, passes through the focal point, hits the mirror, and is then reflected back parallel to the OA; this is essentially the red ray in reverse.

At this point, we can already tell where the image will form. An observer on the left would trace these rays back and think that they originated at point P. This is where the image will form. Let's do two more to verify the result:

- The green ray leaves the arrow head, hits the mirror at its centre, where the mirror is perpendicular to the OA, and the ray is therefor refleced at the same angle with respect to the OA as it was incident. We can generate this ray easily, since it will pass under the object at the same distance from the optical axis as the head of the arrow is above it (see figure for the plane mirror, above).
- The yellow ray leaves the head of the arrow, travels along a radius of the spherical surface, hits the mirror normally, and is therefor reflected back along its own path through the CoS.

At the point at which these rays cross, or in the event that they do not cross, the point at which they <u>seem</u> to have crossed, is where the image will form. The distance from mirror to image is *i*, the *image distance*. If the rays actually cross,as in this example, the image is described as being *real*, but if they only seem to have crossed (example below) the image is called *virtual*. We can define the *magnification* as the ratio of the image size to that of the object, $|M| = h_i/h_o$. If $|M|>1$, we say that the image is *magnified*, but if $|M|<1$as it is here, we say that it is *diminished*. We can also describe the orientation of the image as *upright* (same as the object) or *inverted*, as it is here.

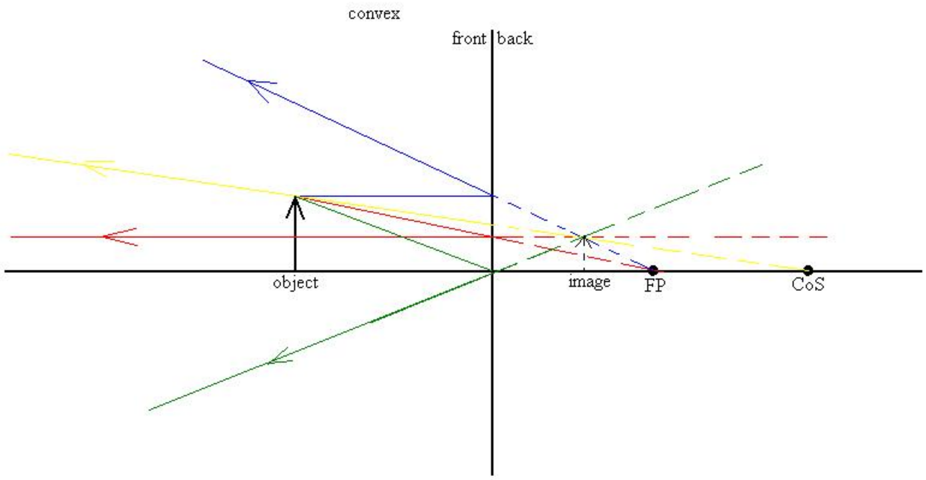
Here is another diagram using a concave mirror, with the same colour code as above:



Note that we cannot draw a line from the arrow head through the focal point, then to the mirror, so we need to use a trick: draw a ray (blue) from the focal point which just grazes the arrow head, continue onward toward the mirror, then reflect it back parallel to the OA. Similarly for the yellow ray, we start at the centre of the sphere, just graze the tip of the object, and continue from there as before.
How would one describe this image?

Answer

Here is an example using a convex mirror:



Here, the rays are constructed a little differently, but with a correspondence to the four described above:

- The blue ray leaves the arrow head, comes in parallel to the OA, then is reflected back away from the focal point.
- The red ray leaves the arrow head, head towards the focal point, hits the mirror, and is then reflected back parallel to the OA.
- The green ray leaves the arrow head, hits the mirror at its centre, where the mirror is perpendicular to the OA, and the ray is therefor refleced at the same angle with respect to the OA as it was incident. We can generate this ray easily, since it will pass under the object at the same distance from the optical axis as the head of the arrow is above it (see figure for the plane mirror, above).
- The yellow ray leaves the head of the arrow, travels towards the centre of the spherical surface, hits the mirror normally, and is therefor reflected back along its own path.

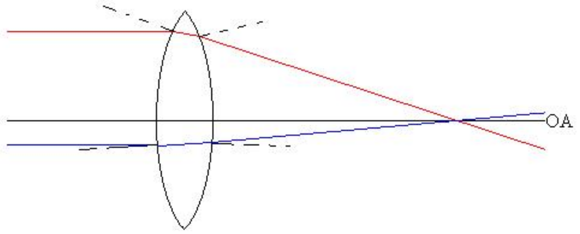
Since these reflected rays do not actuallt cross on the left side o f the mirror, we need to trace the paths back to see where they seems to have crossed, which gives us the location of the image.

Describe the image.

Answer

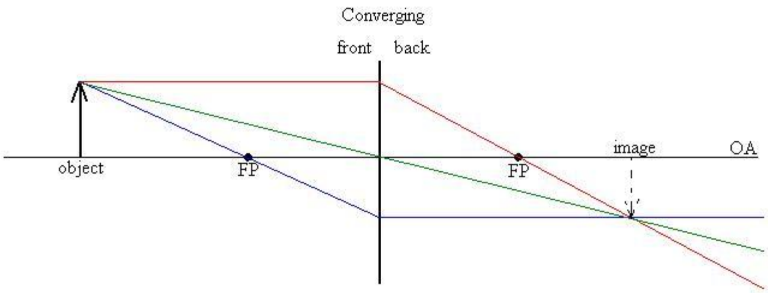
Ray Tracing and Lenses - Converging and Diverging

Now consider a *lens*. Lenses are often described as concave or convex, although this is really incorrect, since lenses have two surfaces. For example, a lens could be double convex, or plano-concave, *et c*. Better to describe them as *converging* or *diverging*. Consider for example a double convex lens, made of glass, sitting in air. Light coming parallel to the OA will hit the first surface and be refracted according to Snell's Law, as shown (The black, dashed lines are the normals at each surface.). It will travel through the glass, then be refracted as shown at the second surface.



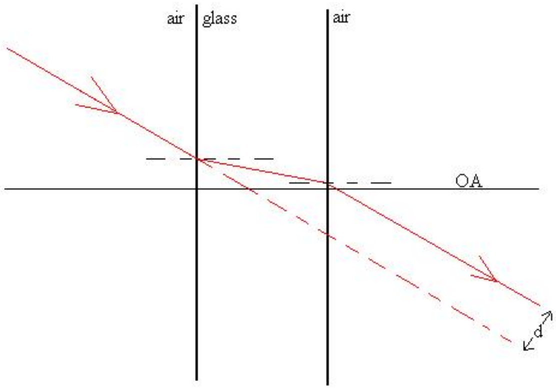
If we draw in several rays, we see that they will all pass through a point on the OA which we shall call (surprise!) the *focal point* of the lens. However, unlike a mirror, a lens will have two focal points, one on each side, since we can always send light in from the other side and get the same effect.
Some types of lenses actually have even more than two focal points, but this is the last time you'll hear about them from me.
There is no reason to think that the focal points on each side will be at the same distance from the lens, however, we will now restrict the discussion to *thin lenses*, for which we shall assert without proof that the focal points are in fact at the same distance from the lens on each side.

Here are the rules we will use for constructing ray diagrams for converging lenses:



One red ray from the tip of the arrow parallel to the OA, through the lens, and bent through the focal point.
One blue ray from the tip of the arrow, through the other focal point, hit the lens, and come out parallel to the OA. This is the the rule for the red ray used in reverse.
One green ray from the tip of the arrow, through the centre of the lens undeflected. This last requires some explanation:

Consider a ray which hits near the centre of the lens at some angle as shown.

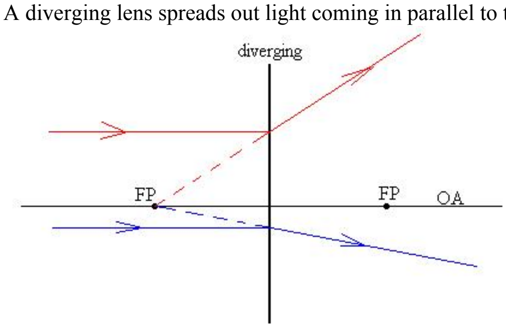


The ray will be refracted at the first surface, which we shall assume is essentially perpendicular to the OA. It hits the second surface, which is also perpendicular and exits in the same direction as it was initially traveling in. Why?
$n_{air} \sin \theta_{air,left} = n_{glass} \sin\theta_{glass, left} = n_{glass} \sin\theta_{glass, right} = n_{air} \sin \theta_{air,right}$
Note, however, that the ray is offset from its original line of travel, but as the thickness of the lens goes toward zero, the amount of offset (d) also decreases. So, we conclude that for a thin lens, the ray through the centre is undeflected.
Describe the image produced above.

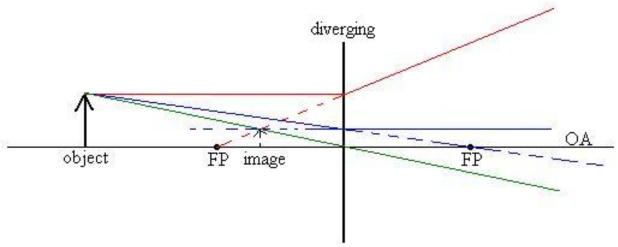
Answer

Such an arrangement might be seen in a simple *camera*, with the image projected into a film.

A diverging lens spreads out light coming in parallel to the OA, so that it seems to have come <u>from</u> the focal point:



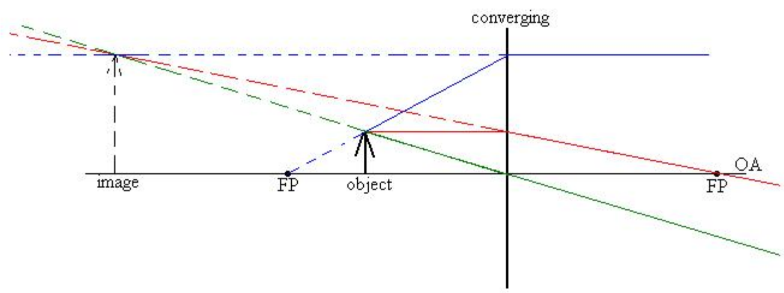
Here is an example of ray tracing for a diverging lens:



One red ray from the tip of the arrow parallel to the OA, through the lens and bent away from the focal point.
One blue ray from the tip of the arrow, towards the other focal point, hit the lens, and come out parallel to the OA.
One green ray from the tip of the arrow, through the centre of the lens undeflected.
Describe this image:

Answer

Let's try another converging lens, where the object is inside the focal length:



One red ray from the tip of the arrow parallel to the OA, through the lens and through the right hand focal point.
Blue ray: here, we had to use our trick again, starting from the left hand focal point, just grazing the tip of the arrow, hit the lens, and come out parallel to the OA.
One green ray from the tip of the arrow, through the centre of the lens undeflected.
Describe this image:

Answer

This is how a magnifying lens works.

Question:
Occasionally, students will ask, 'Which ray (of the many) from the object will reach the image first?'

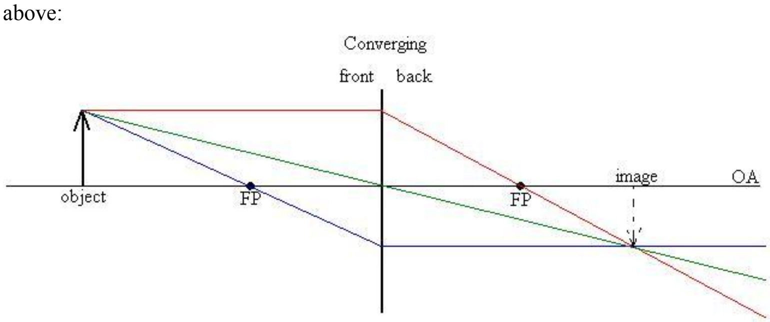
Answer

How is that possible? Look at figure 8f18 above. Clearly the red and blue paths are longer than the green path.

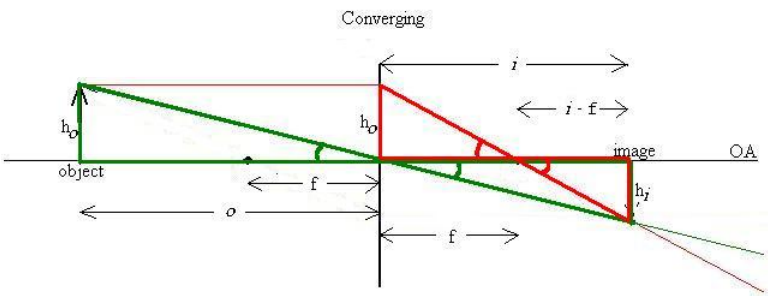
Answer

The Thin Lens Equation

Now that we understand what's happening, we should probably put this discussion into a more mathematical framework. Consider one of the cases above:



To review, *f* is the focal length, the distance from the centre of the lens to the focal points (marked with dots). The object is distance *o* from the lens, the image distance *i* from the lens. The object's size is h_o, and that of the image is h_i.



Now consider the green triangles; they are both right triangles and the marked angles are equal, and so they are *similar*. We can make some statements about the ratios of the sides of similar triangles, namely that
$h_i/h_o = i/o$.
Now, consider the red triangles, which are also similar to one another, and so we can write that
$h_i/h_o = (i - f)/f$.
So,
$(i - f)/f = i/o$
$(i/f) - 1 = i/o$
$1/f - 1/i = 1/o$
$1/o + 1/i = 1/f$
This last is the *thin lens equation*, which shows the relation among image distance, object distance, and focal length. I will assert without proof that this will work for all thin lenses and for all spherical mirrors, provided certain sign conventions are followed. For example, we assumed that all distances in the preceding derivation were positive. To make this work in all cases, use the conventions below:

<table>
<tr><td colspan="2">Assume that the light reaches each optical element from the left.</td></tr>
<tr><td><u>Lenses</u>
converging $f > 0$ diverging $f < 0$ flat f = infinity
object on the left $o > 0$ (real) object on the right $o < 0$ (virtual)
image on the left $i < 0$ (virtual) image on the right $i > 0$ (real)</td><td><u>Mirrors</u>
concave $f > 0$ convex $f < 0$ plane f = infinity
object on the left $o > 0$ (real) object on the right $o < 0$ (virtual)
image on the left $i > 0$ (real) image on the right $i < 0$ (virtual)</td></tr>
<tr><td colspan="2">We can also re-define the magnification as $M = - i/o$. In this way, the following information can be gleaned:
If $M > 0$, then the image is upright If $M < 0$ then the image is inverted
If $|M| > 1$ the image is magnified If $|M| < 1$ the image is diminished</td></tr>
</table>

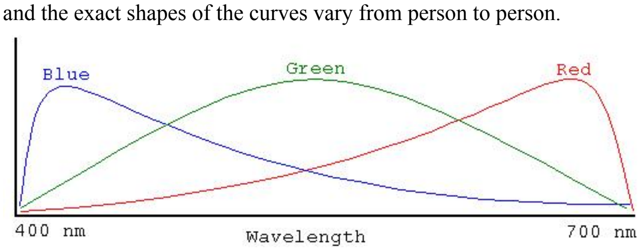
Let's briefly consider the meaning of having a 'virtual object.' Suppose that there are two (or more) lenses or mirrors through which light must pass. Where will the final image form? What we do is consider each optical element separately. Find the image produced of the actual object by the first optical element, then use that as the object of the second element. In some cases the image of the first element will form behind the second element, and so we consider it to be a virtual object.

Defects of the Eye

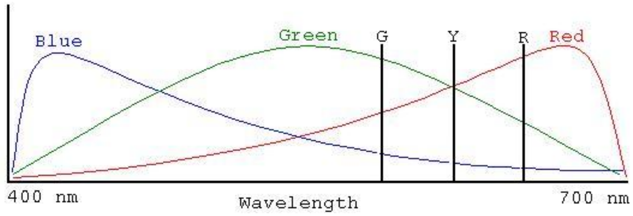
In normal operation, the eye behaves much like a camera in that a converging lens focusses a real image onto the photosensitive surface at the back of the eye, the *retina*. Since the image distance is essentially fixed, focussing is accomplished though muscles in the eye which deform the flexible lens, changing the focal length, *f*; this is called *accomodation.* The normal eye can focus comfortably on objects from infinity to about 25 cm away; this nearest distance is called the *near point* of the person's vision. The amount of light entering the eye is adjusted with the *iris*, a membrane which surrounds the optical opening or *aperture* into the eye, the *pupil*; it also has an effect on the eye's resolution, which we'll discuss in Section 9. The eye is covered with a transparent membrane called the *cornea*, which also acts to refract light.

The retina has two basic types of photoreceptors:

Cones, which are concentrated near the center of the retina (the *fovea*), appear to be able to distinguish among three basic colors (red, green, and blue, each with some overlap into neighboring parts of the spectrum), but which are not sensitive to low light levels. The figure is very qualitative and the exact shapes of the curves vary from person to person.



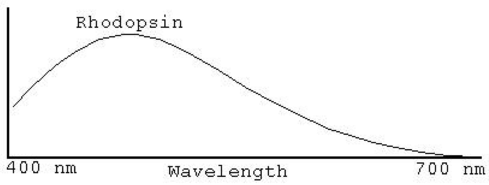
It is thought that light of any <u>single</u> given wavelength will excite all three types of photo-receptors in a unique combination; however, the correct combination of several wavelengths can produce the same sensation as would a single wavelength not actually present! For example, consider the eye's response to a single wavelength of yellow (589 nm), which excites equal amounts of green and red, plus a little blue, then consider a combination of a red wavelength (670 nm) and a green wavelength (535 nm) which excites the eye receptors in the same proportions; that combination will be perceived as yellow, even though no yellow light is actually present.



A second example: Light of just two wavelengths, 480 nm and 580 nm, can stimulate the same response as a complete spectrum of white light.

In fact, this is how color televisions work, by using mixtures of three basic or *primary colors* to evoke in the human eye the sensation of colors which are not actually present.
Lastly, since the actual response curves can vary from person to person, it is entirely possible that one person's orange could look very different from another person's orange.

Rods, which are very sensitive to low levels of light, are more evenly distributed over the retina, but do not distinguish between different colours of light. The photosensitive chemical in the rods is *rhodopsin*.



In low light levels, humans lose their color perception (Try it! Go out before dawn, and everything is in black and white!), since only the rods are receiving enough light to trigger signals to the brain; at higher light levels, the rods shut off and the cones take over.
Normally, in going from a bright room to a dark room, there is a time delay until the cones shut off and the rods turn back on. The poor response of rhodopsin in the red part of the spectrum allows humans to preserve their *night vision* by using only red light while still indoors. Since the rods' response is not saturated by the bright red indoor light, the rods can begin seeing in the dark almost immediately.

It is thought that photo-chemical reactions in these cells initiate signals sent along nerves to the brain for processing. While most nerve connections for each side of the body are sent to the opposite side of the brain, vision signals from each eye are sent to both sides of the brain.

Color Blindness

There are in fact many types of *color blindness*, all of which seem to occur almost exclusively in males (although under certain conditions, almost anyone can be rendered temporarily color blind) and for many different apparent reasons. A good introduction to the subject is in Falk, Brill, & Stock, Seeing the Light, John Wiley & Sons, New York (1986). All such defects seem to be able to be attributed to deficiencies in the photodetectors in the eye. Very few people (<0.003%) are literally color blind in the sense of being completely unable to distinguish colors. More commonly, one type of cone receptor doesn't work or its response is shifted in wavelength so that it overlaps with another type's response. A common example of such a situation is *red-green color blindness*, in which certain shades of red appear identical to certain shades of green. This usually isn't a problem, except when driving in the Tipperary Hill section of Syracuse (a historically Irish district), which has a traffic light with green on top and red on the bottom; about once a year, one or another color blind driver causes an accident because he thought he had the green.

Myopia

Myopia (= 'shut-eye-ness' from the Greek *muo+ops,* since myopic people tend to squint) is also known as *near-sightedness* or *long-eye*. Symptoms include an ability to focus on very nearby objects (great if you're a watchmaker or tailor, and as this condition runs in families, so too did these professions) and an inability to focus on distant objects. Generally, the cornea is too thick and makes the focal length of the lens/cornea combination too short, so that the images are focussed in front of the retina. If you are near-sighted, you can test this by gently pushing in on the front surface of your eyeball; the image you see should become much clearer. Occasionally, the condition is caused by an abnormal elongation of the eyeball, so that the retina is placed too far back from the lens; this condition is usually associated with increased risk of retinal separation, which can be treated by laser or liquid nitrogen welding of the retina to the back of the eye. The usual treatment for myopia is to place a diverging lens in front of the eye to increase the effective focal length, but more recent treatments include *radial keratotomy*, in which slits are cut in the cornea to allow it to flatten out, thus reducing its optical 'power,' and *laser abblation* of the cornea, which reshapes it. Squinting reduces the size of the eye's aperture and, through a process not explained here, improves the eye's ability to focus on more distant objects.

Hyperopia

Actually *hypermetropia* (= 'eye-ness beyond measure' from the Greek *hupermetros+ops*), *hyperopia* is also known as *far-sightedness*. The symptoms of hyperopia include an ability to see well at a distance, but an inability to focus on nearby objects. In this situation, the cornea/lens system has too long a focal length, and the image typically falls behind the retina. Correction usually involves placing a converging lens in front of the eye.

Presbyopia

Presbyopia (= 'old man-eye-ness' from the Greek *presbus+ops*; remember that the Presbyterian Church is run by elders) is most common in older people, and is due to a reduction in the flexibility of the lens, thus making it more difficult to focus at both far and near distances. Correction involves the use of a *bifocal* lens (invented by Franklin) with a diverging lens on top for viewing distant objects, and a converging lens on the bottom for reading and other close work.

Astigmatism

Astigmatism (= 'not-(forming)-a-point' from the Greek *a+stigma(t)+ismos*) is a defect where the cornea is not symmetric, causing the eye to have different focal lengths for light rays in one plane than for rays in some other plane. The result is that a point source of light is not focussed to a point on the retina, but instead into some elongated shape. Correction involves placing a cylindrical lens in front of the eye, which affects only the light in certain planes, or the use of contact lenses, which present an artificial symmetric surface at which the light is first refracted.

Question:
Recall the book or films of The Lord of the Flies by William Golding. In that story, Piggy's glasses were coveted by both tribes of boys as a tool to start fires. What flaw is there in this plot device?

Answer
