

Section 2-9 - Wave Optics

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Wavelength of Light in a Material

Now, we need to go back to thinking about light as a wave, rather than just as a ray. In the previous section, we showed how the speed of light is reduced in a material from that in a vacuum by a factor n , the index of refraction:

$$v = c/n.$$

Remember from previous discussions that, for any wave,

$$v = f\lambda.$$

So, as light enters a material with a different index, either f or λ or both must change as the speed changes. We developed the idea of frequency as the number of wave crests passing a given spot per unit of time. If we consider light waves passing through some interface, it seems clear that the number of crests incident on the one side must equal the number exiting on the other, else they would 'pile up' at the interface like cars in a traffic jam. So, the frequency f does not change from material to material. Clearly, then, we can write that

$$\lambda' = \lambda_0/n,$$

that is that the wavelength of light in a material with index of refraction n is given by the light's wavelength in vacuum (λ_0) divided by n .

Young's Double Slit Experiment

Thomas Young was a pretty remarkable fellow. In [Great Experiments in Physics](#), Morris Shamos details his accomplishments:

- Practicing physician (!)
- Egyptologist (both archaeologist & philologist)
- Work in optics
- Developed the notion of kinetic energy and defined work as a force acting through a displacement, Work-Energy Theorem
- Elastic properties of materials (Young's modulus and the like)
- Theory of tides
- First to postulate presence of three color receptors in the human eye

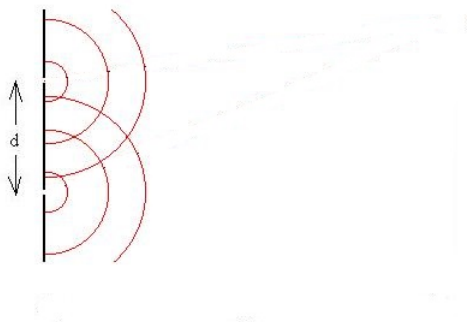
All that and he died at age 56. What's the old joke? When Mozart was my age, he'd already been dead ten years.

One experiment which really swayed the scientific community toward the wave model was *Young's Double Slit Experiment*. Young took a barrier with two rectangular openings (or slits) and illuminated them with light of almost a single wavelength. If light were a particle, as Newton suggested, Young would have expected to see two rectangular bright patches when he opened both slits, much the same pattern as if you were to throw spitwads through two openings. What he actually found was that he did not get simply two rectangles, but instead a complicated pattern of varying light and dark. Since he had the advantage of already knowing that these *interference* effects are present for sound, he deduced that this strange phenomenon could only occur if light was a wave, and in fact, in a speech at the Royal Institution, he dared the Newtonians to explain it using their corpuscular theory.

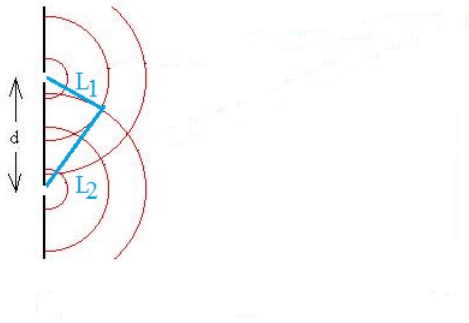
First, we'll do an accurate analysis, then we'll do a derivation that is a bit closer to what's in your book.

Consider a barrier with *monochromatic light* of wavelength λ incident normally. Two slits (seen here in profile) are spaced a distance d apart. The light passes through the slits (let's assume that they are infinitely narrow) and travels from each slit and eventually impinge on a screen a distance D away.

We can now invoke *Huygen's Principle* to assert that the slits can be treated as two sources of new waves, which in this case start out in phase with one another.



Let's follow these two waves along rays to a specific spot in the space to the right of our barrier.

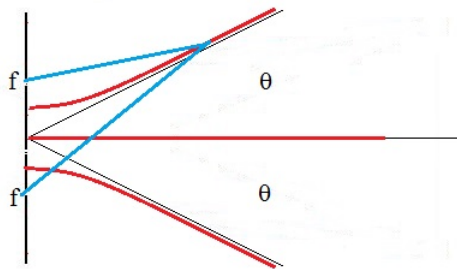


These waves will arrive at our point in phase if the difference between the lengths of the paths taken is a integer number of cycles, or in this case, wavelengths: $|L_1 - L_2| = m\lambda$. Such a condition when the two waves add together is called *constructive interference*.

Can we find the set of points where constructive interference occurs? Well, the condition set forth above is exactly the definition of a *hyperbola*. So, let's review.

Start with two special points called the *foci*, separated by a distance $2c$. A hyperbola is the set of points such that the difference in the distances from each such point to each of the two foci is equal to $2a$. The *eccentricity* of the hyperbola is given by $e = c/a$, and the equation for the curve is $y^2/a^2 - x^2/b^2 = 1$, where $b = (c^2 - a^2)^{1/2}$. In our situation, $2a = m\lambda$ and $2c = d$, the separation between the slits. Substitution and some re-arranging results in

$$y = \pm m\lambda (1/4 + x^2/(d^2 - m^2\lambda^2))^{1/2}.$$



The red lines in the figure represent possible $m = 0$ and $m = 1$ curves.

Now, generally, we don't look at the light too close to the barrier, but instead we look at what happens on a screen some distance away. Remember that the arms of a hyperbola approach lines called *asymptotes*. The angle theta these asymptotes make with the central line is given by $\sin \theta = 1/e$.

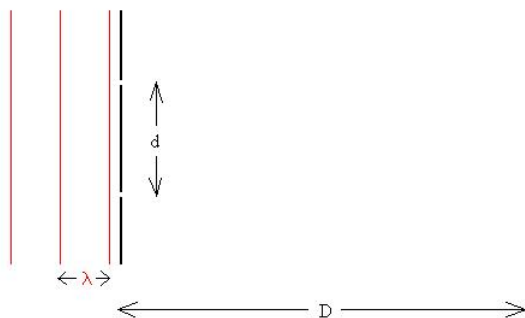
So, substituting,

$$\sin \theta = 1/e = 1/(c/a) = 2a/2c = m\lambda/d$$

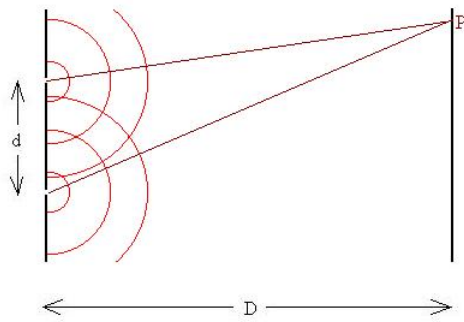
or,

$$d \sin \theta = m\lambda. \text{ with } m = 0, \pm 1, \pm 2, \text{ et c. Note, however, that there is a maximum value for } m, \text{ since the sine function can not exceed one in value.}$$

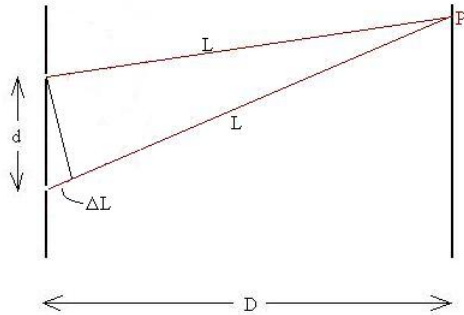
OK, now let's do this in a more conventional way. Place a screen a distance D from the barrier



Again, we assert that the slits can be treated as two sources of new waves which start out in phase with one another. Let's follow these two waves along rays to a specific spot P on the screen,



at which point they will have a particular phase relationship due to the difference (ΔL) in the lengths of the path each ray took.



If the difference ΔL is an exact integer multiple of the wavelength of the light,

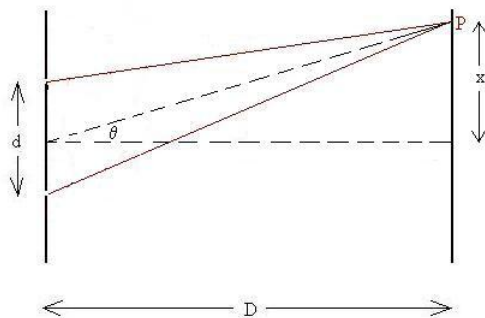
$$\Delta L = m\lambda,$$

the waves will arrive in phase, and add together to produce a bright spot on the screen (*constructive interference*), but if the difference is an integer plus one-half times the wavelength

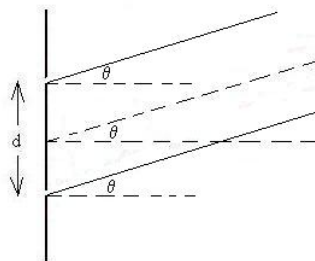
$$\Delta L = [m + \frac{1}{2}]\lambda,$$

they will arrive out of phase by 180° , and they will cancel each other resulting in a dark spot on the screen (*destructive interference*). Intermediate phase differences will result in dim, but not dark, spots.

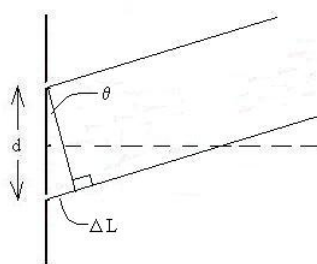
Now, if we assume that $D \gg d$, we can make a couple of assumptions which will allow us to develop a nice relationship. Let Point P be a distance x from the line which perpendicularly bisects the line connecting the slits. Let the angle θ be that angle from the bisector over to the line directed toward Point P .



If $D \gg d$, then the line to P and the two path lines are approximately parallel, that is, they all make the same angle θ with the bisector:



The line we drew earlier between the two rays is now perpendicular to each of them, and by geometry, makes the same angle θ with the barrier:



Using a little geometry and trigonometry, we see that

$$\Delta L = d \sin \theta.$$

However, we already know that to produce a bright spot,

$$\Delta L = m\lambda,$$

so,

$$d \sin \theta = m\lambda.$$

We expect that there will be a number of such angles, so we'll modify this to

$$d \sin \theta_m = m\lambda; \quad m = 0, \pm 1, \pm 2, \dots$$

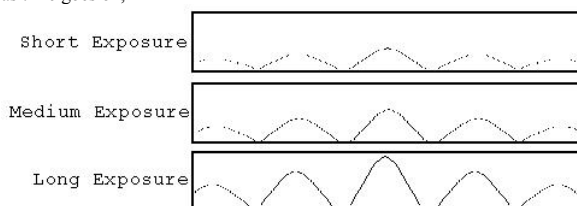
A similar calculation for dark spots gives

$$d \sin \theta_m = [m + \frac{1}{2}]\lambda; \quad m = 0, \pm 1, \pm 2, \dots$$

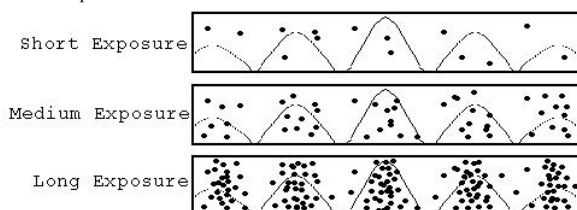
Question: What pattern will be seen if the two sources had been initially 180° out of phase?

As an approximation for small angles, we see that $\tan \theta = x/D \sim \sin \theta$.

How do we now, in the twenty-first century, reconcile this 'proof' of the wave nature of light with Einstein's 'proof' of the particle nature of light? We find that, if we shine a very low level light on a pair of slits and record the pattern that forms on a photographic plate, that the film does not register a faint diffraction pattern which gets clearer as time goes on,

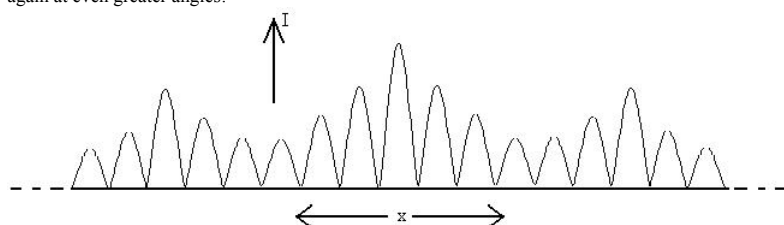


but instead it records a series of random 'hits' of photons such that the intensity curve we expect to see represents the statistical frequency with which the photons hit in different spot.



So, even in this single experiment, we can see evidence of both the wave and particle natures of light!

Now to answer a common question: Why do the maximums get fainter the farther they are from the central lines? One reason is that the outlying points are simply farther from the slits, and we saw in our discussion of sound that the intensity of a wave falls off with distance from the source. A second reason is due to the fact that the slits are not infinitely thin, as we supposed in the derivation. Factoring this effect in explains why the peak intensities fall off, and also how they can actually become brighter again at even greater angles!



However, that effect is too complicated to explain here.

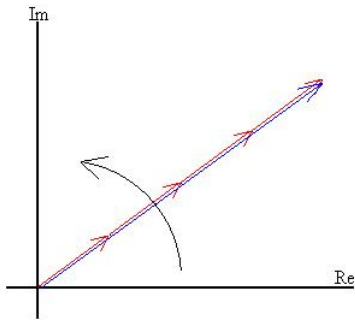
Multiple Slits

Now, what if there had been many (say, N) slits, all very thin and separated from their nearest neighbours by d ? Certainly, we can see that the very brightest spots will be formed when all of the waves arrive in phase, which of course means that rays from adjacent slits are in phase, and we've already done that calculation:

$$d \sin \theta_m = m\lambda; \quad m = 0, \pm 1, \pm 2, \dots$$

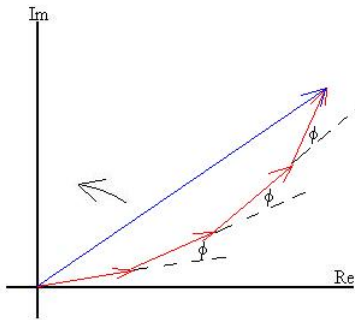
But, we can certainly imagine a situation where only a majority of waves arrive in phase together, resulting a smaller maximum. This will involve much trigonometric calculation, even for low N . But we can once again make use of the *phasor*. Draw an arrow with a length proportional to the amplitude of the light wave from each source

(in red; we'll assume them to be equal) and with a direction representing the phase. For example (let $N = 4$), if all waves arrive in phase, we might draw this:



with the blue phasor representing the result.

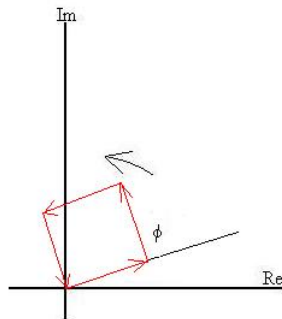
Now, suppose that we move along the screen so that the waves do not quite arrive in phase, due to the slightly different path lengths travelled by each ray:



where we see that the amplitude of the sum of the four waves is reduced. The angle ϕ can be shown to be $\phi = 2\pi \Delta L / \lambda$, that is, when $\phi = 2\pi$, $\Delta L = \lambda$ and the arrows all line up again, and ΔL we already showed for adjacent slits to be $\Delta L = d \sin \theta$.

So, $\phi = 2\pi d \sin \theta / \lambda$.

Now, go back to the diagram above. As we increase ϕ , the total amplitude will decrease, until (for four slits) $\phi = \pi/2$ or 90° :

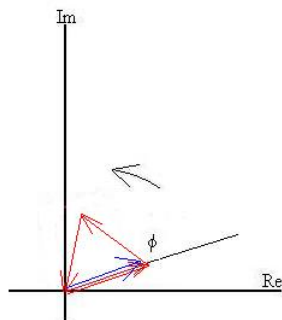


So, the first minimum occurs when

$$d \sin \theta = \frac{1}{4} \lambda.$$

We can even generalize that result, since the diagram could just as well represent the case of $\phi = 360^\circ + 90^\circ$, or $\phi = 2 \times 360^\circ + 90^\circ$, to $d \sin \theta = [m + \frac{1}{4}] \lambda$.

The next maximum occurs when $\phi = 2\pi/3 = 120^\circ$:



or, when

$$d \sin \theta = \frac{1}{3} \lambda,$$

and more generally, when

$$d \sin \theta = [m + \frac{1}{3}] \lambda.$$

Note that this maximum will be much less bright than the last one; the amplitude is one fourth, and so the intensity is one-sixteenth.

The next minimum occurs when each successive arrow points 180° from the previous one (not shown), so that $\phi = \pi$ and

$$d \sin \theta = \frac{1}{2} \lambda,$$

or, more generally, when

$$d \sin \theta = [m + \frac{1}{2}] \lambda.$$

The next maximum occurs when $\phi = 4\pi/3$ or 240° ; the phasors would form another equilateral triangle, although flipped the other way.

$$d \sin \theta = \frac{2}{3} \lambda,$$

or, more generally,

$$d \sin \theta = [m + \frac{2}{3}] \lambda.$$

The next minimum will occur when $\phi = 270^\circ$ or $3\pi/4$, such that

$$d \sin \theta = \frac{2}{3} \lambda,$$

or, more generally,

$$d \sin \theta = [m + \frac{2}{3}] \lambda.$$

Then, we're back to where we started with $\phi = 360^\circ$, with the waves all in phase again.

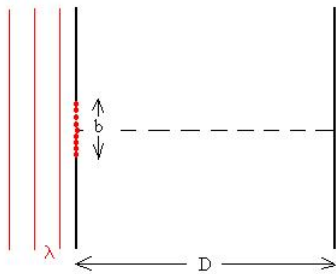
A common situation involving many thousands of slits is the *diffraction grating*, used in some spectrometers. In that case, there are *principal maxima* which meet the condition

$$d \sin \theta_m = m\lambda; \quad m = 0, \pm 1, \pm 2, \dots$$

and many, many secondary maxima which are many times dimmer than the principal ones, so dim that they can usually be ignored. The advantage of using a grating instead of a double slit is that the principal maxima become extremely narrow, allowing for easy resolution of light from two nearly identical wavelengths of light.

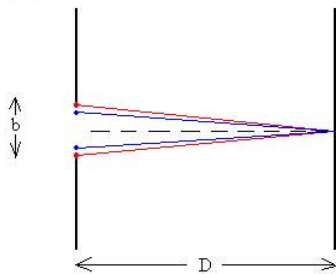
Single Slit Diffraction

What happens if light of a single wavelength is incident perpendicularly on a single slit of width b ?



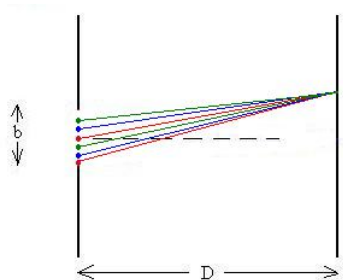
In that case, we have an infinite number of sources (some shown in red), all in phase with one another. Let's examine just a couple of very special cases.

Consider what happens at the point on the screen right on the perpendicular bisector. Look at the point sources in pairs, starting with the two end points (in red).

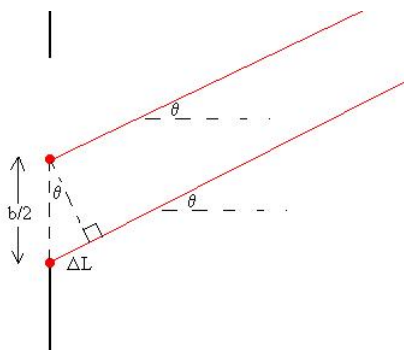


Since the distances from each to the point of observation are the same, these two waves should arrive in phase with each other. Now, let's look at the two points just infinitesimally toward the centre of the opening (in blue). The distances traveled by the two rays originating there will also be equal, so that those waves arrive in phase with each other, and only slightly out of phase with the first pair we looked at. Now, continue to look at these rays pairwise; each arrives in phase with its 'pair-mate,' and if $D \gg b$, almost in phase with all the rest of the rays. So, we should expect a nice bright maximum at that centre point (at $x=0$ or rather $\theta=0$).

Let's consider the first minimum or dark spot formed as the angle θ increases. At this point, all the waves must cancel out. We might assume that we can once again use an accounting trick to figure out the conditions necessary, and we might be right. This time consider the point sources in pairs, but separated by distance $b/2$, as shown in the figure (match the colours):



Let's assume again that $D \gg b$, so that the rays are all approximately parallel. Now consider the two rays marked off in red:



The angles they make with lines parallel to the bisector line are now both θ , as is the angle marked in the little triangle. Using some trigonometry, we see that $\Delta L = [b/2] \sin\theta$.

We also recall from the discussion above that any difference in phase in the waves as they arrive at the screen must be due to the path length difference, ΔL ; in this case, we are considering the first minimum, so

$$\Delta L = \lambda/2,$$

which tells us that

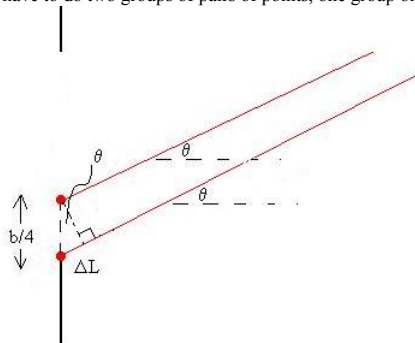
$$\Delta L = [b/2] \sin\theta = \lambda/2,$$

or

$$b \sin\theta = \lambda.$$

Note the similarity in this relationship to the requirement for the first maximum in the double slit case. Now, we've shown that if this requirement is met, these two rays will cancel. But as we consider all of the other pairs of rays, each separated from its 'pair-mate' by $b/2$, we can see that all of the rays will cancel in pairs, so this will indeed be a minimum or dark spot on the screen.

Now, that's just the first minimum, how about the others? Let's repeat the derivation above, but instead of considering pairs of points separated by $b/2$, let's do $b/4$ (we'll have to do two groups of pairs of points, one group on one side of the centre line and one group on the other side).



Then,

$$\Delta L = [b/4] \sin\theta = \lambda/2,$$

or

$$b \sin\theta = 2\lambda.$$

For the next minimum, divide the point sources into three groups of pairs of point sources separated by $b/6$. Then

$$\Delta L = [b/6] \sin\theta = \lambda/2,$$

or

$$b \sin\theta = 3\lambda.$$

We surmise through *induction* that the minimums occur when

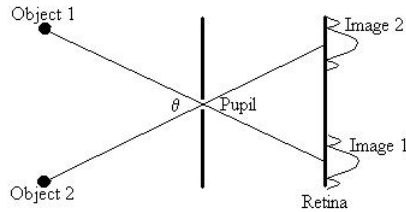
$$b \sin\theta_m = m \lambda, \quad m = +1, +2, +3, \dots$$

Unfortunately, the conditions required for the maximums or bright spots (other than the central one at $\theta = 0$) are not so easy to calculate; they are only approximately midway between the minimums, but not exactly.

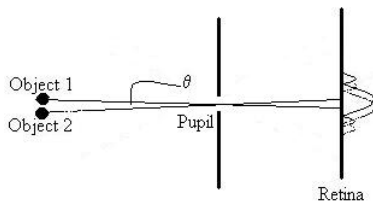
Circular Apertures and Resolution

Since many optical devices have circular openings rather than rectangular ones (cameras, eyeballs, *et c.*), it seems that we should spend some time discussing the diffraction from a circular aperture. We saw previously that a rectangular aperture will produce a central maximum with an angular half width of $\sin \theta \sim \lambda/b$, with a number of secondary maximums beyond that angle in both directions. What we find for the circular aperture of diameter D is that there will be a central bright disc with an angular radius of $\sin \theta \sim 1.22\lambda/D$ with secondary maximums at larger radii from the centre.

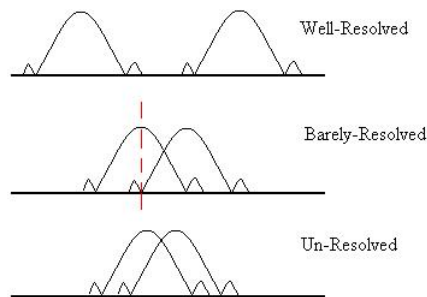
Often, we are concerned whether we can distinguish the images formed by two objects which lie on nearby sight lines. For example, two stars, even though we can consider them to be point sources, will produce images on our retinas which have some non-zero size, due to the diffraction through our pupils (here, only the first few maximums are shown for illustration).



If the images overlap sufficiently, we will not be able to distinguish them, and we will see one big blob.



Often, we use *Rayleigh's Criterion* to decide whether the images overlap too much: if the central maximum of Image One is closer to the the central maximum of Image Two than is the first minimum of Image Two, then the images are not resolvable.



It follows then that, for the images to be resolved,
 $\sin \theta \sim \theta_{\text{radians}} \sim 1.22\lambda/D$,
 where D is the *aperture* (diameter of the opening).

Example: You're out on a flat dark desert and a car is coming toward you. Estimate the minimum distance at which you can resolve the car's two headlights. HINT: Let's guess that the headlights are $x = 1\text{ m}$ apart (a small car) and that the distance to the car (d) is large compared to that. Then, we could say that the tangent of θ is about x/d , which is also about θ itself, if expressed in radians. Also, assume that the wavelength of the light is about 550 nm . The eye's aperture on a dark night, maybe 5 mm ?

Answer

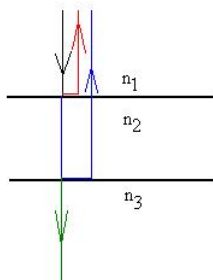
Note that for eyes, there is another consideration to take into account. The cornea has a fairly high index of refraction (~ 1.5); according to Snell's law, the angle of a ray entering the eye is deflected toward the eye's optical axis by some amount before being diffracted by the pupil, so that the objects to be resolved by the eye must actually be spaced further apart than the equation above suggests. A mitigating factor, though, is that the wavelengths of light passing through the fluids in the eye are shortened by a factor of 1.34 (the index of the fluids), so that the widths of the images are reduced and then so is the minimum angle to meet the Rayleigh criterion.

Example: Consider a 2.2 metre diameter camera in a KH-12 spy satellite in orbit 200 km above the earth. Assuming no atmospheric degradation of vision, what is the smallest separation between two objects on the earth's surface this camera can resolve?

Answer

Interference in Thin Films

Let's discuss another example of interference. Consider the effect seen in water puddles in the parking lot after a rain. Often, the water will appear to be different colours, almost like a rainbow. This is caused by interference of light reflected from the thin film of oil floating on the water. Let's consider such a film of thickness d and index of refraction n_2 atop another material of index n_3 . Light of wavelength λ_0 in vacuum travels from above (let's restrict ourselves to the special case of normal incidence) vertically through a material of index n_1 and is incident on the upper interface. In Section 1-14, we saw that mechanical waves were reflected with phase inversion if $Z_2 > Z_1$ and without if $Z_2 < Z_1$. A similar result is observed with light waves (See [note](#) at end of section): The phase of the electric component of the reflected wave is inverted on reflection if $n_2 > n_1$, and it is not inverted if $n_2 < n_1$. So, the ray is partially reflected from the first interface with a phase change of either 0° or 180° which we'll represent by

PC_{top}

Note that the reflected ray has been drawn schematically with a slight horizontal offset to aid visualization; in fact, it passes back up along its original path. The rest of the ray continues down into Material 2, where it eventually hits the second interface, at which point some continues down into Material 3 and some is reflected back upward (with a phase change PC_{bottom} of either 0° or 180°) and passes back up into Material 1. We shall ignore the multiple reflections which actually occur. It is these two rays we will concern ourselves with: the first reflection from the top interface (red) and the light from the first reflection at the second interface (blue). If these two rays are in phase, we will see a bright reflection, and if they are completely out of phase, there will be no reflection. Also, note that there is also some light transmitted through into Material 3.

FIGURE

We can tentatively write that

PC_{top} + PC_{bottom} = an integer number of cycles (constructive interference or bright reflection)
 an integer number of cycles plus one half cycle (destructive interference or no reflection).

Now, this result isn't quite right, because there is another consideration which contributes to the phase difference between the rays. Remember that in our discussions of the single and double slit situations there was a phase difference introduced when one wave traveled a longer distance than the other did. How much farther does the ray reflected from the lower interface travel than does the one reflected from the top?

Answer

So, we should change this to:

PC_{top} + PC_{bottom} + 2d = an integer number of cycles (constructive interference or bright reflection)
 an integer number of cycles plus one half cycle (destructive interference or no reflection).

We see, though, that there is now a dimensional mismatch, in that the PC terms and the right hand side (measured in degrees or cycles) are dimensionless and the distance term (2d) has dimension [Length]. We'll fix that by using the wavelength of the light in material two (λ_2) as the distance equivalent of a cycle, inserting 0 for 0° and $\lambda_2/2$ for 180°. We need to use λ_2 because we need to know how many extra wavelengths (cycles) are included in the distance 2d inside material two.

PC_{top}(0 or $\lambda_2/2$) + PC_{bottom}(0 or $\lambda_2/2$) + 2d = $m\lambda_2$ (constructive interference or bright reflection)
 $[m + 1/2]\lambda_2$ (destructive interference or no reflection).

It's probably more convenient to write this relationship in terms of the wavelength in vacuum: $\lambda_2 = \lambda_o/n_2$. Making this substitution and multiplying through by n_2 gives our final result:

PC_{top}(0 or $\lambda_o/2$) + PC_{bottom}(0 or $\lambda_o/2$) + 2n₂d = $m\lambda_o$ (constructive interference or bright reflection)
 $[m + 1/2]\lambda_o$ (destructive interference or no reflection).

Question:

Why do the lenses of the more expensive cameras appear to be purple (actually magenta)?

Answer

Example:

Suppose that a thin film of oil (index = 1.2) floats on top of water. Light of wavelength $\lambda_o = 600$ nm is incident normally on the surface and produces a very bright reflection. How thick is the oil film?

Answer

We have a bright reflection, so let's assume constructive interference:

$$PC_{top} + PC_{bottom} + 2n_2d = m\lambda_o$$

Since $n_2 > n_1$ and $n_3 > n_2$, there are 180° phase changes (write $\lambda_o/2$) at each interface:

$$\lambda_o/2 + \lambda_o/2 + 2n_2d = m\lambda_o.$$

Here's a neat trick: the two half lambdas add to one whole lambda, which represents a complete cycle. But phase-wise, a difference of a complete cycle is the same as a difference of zero. So, in spite of being added, those two terms can actually cancel. We now have:

$$2n_2d = m\lambda_o.$$

Then,

$$d = m\lambda_o/2n_2 = m*600/(2*1.2) = m*250 \text{ nm}.$$

The case $m=0$ corresponds to there being only one interface and no oil layer; there will be a fairly bright reflection but no interference. The case $m=1$ is the thinnest such oil film which meets our requirements, but other thicknesses ($m>1$) will also work.

Example:

What wavelengths of light will produce bright reflections from a soap-water film of thickness 500nm and index of refraction 1.4?

Answer

Assume that the film has air on each side, so that $n_2 > n_1$ but $n_2 > n_3$. Then,

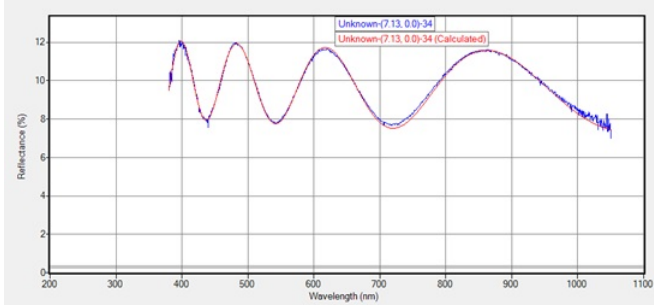
$$\lambda_o/2 + 0 + 2n_2d = m\lambda_o.$$

$$2n_2d = [m - 1/2]\lambda_o.$$

$$\lambda_o = 2n_2d/[m - 1/2].$$

The case $m = 0$ doesn't make sense, so go to $m=1$, and so on.

Please note that this way of looking at the situation is very different then the way presented in your text, which probably asks you to memorize different formulas for different situations. This method, although slightly cumbersome, works in all cases.



Reflection Spectrum of a sapphire film on Gorilla Glass.

Polarization

Now, let's return to the original picture of light as an electro-magnetic wave which was introduced at the start of Section 8. We had mentioned that the electric field could oscillate in different directions for any given direction of propagation.

FIGURE

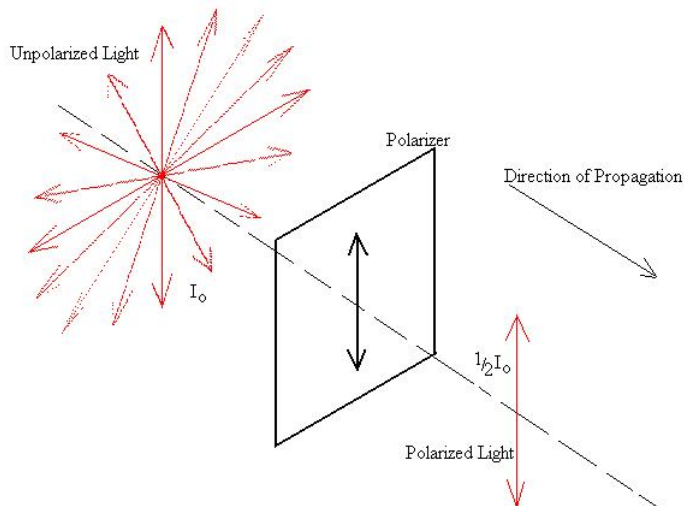
Polarization of light can be accomplished in a number of ways.

- EM waves can be generated by accelerating electrical particles. High energy electrons orbiting in a magnetic field give rise to *synchrotron radiation*, which is polarized. Low energy electrons give rise to *cyclotron radiation*, which is also polarized.
- Reflecting light off of a surface at just the right angle (the *Brewster's angle*: $\tan\theta_B = n_2/n_1$). We'll get back to this later.
- Passing the light through a *polarizer*, as invented by Land in the '30s.

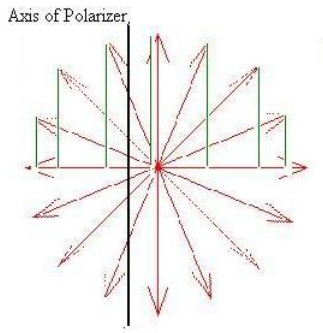
Let's discuss this last. A commercial polarizer consists of a plastic with many long, thin *polymer chains* aligned parallel to one another. Treating the chains with certain chemicals renders them electrically conducting. Electrons in each molecule can then move very easily along the chains, but not easily perpendicularly to the length of the chain. When light is incident on the molecules, the components of the electric fields parallel to the chains make the electrons move up and down the chains and are thus absorbed, while the electric field components oriented perpendicularly to the chains are not absorbed and so pass through. An analogy to this might be a string passing between the slats in the back of a kitchen chair; a wave with displacement in the vertical plane will pass through the chairback unhindered, but one with displacement in the horizontal plane will be stopped.

An *ideal polarizer*, one made of a material with no intrinsic absorption, will pass 50% of an initially unpolarized beam (see [note](#)) of light:

$$I = 1/2 I_0 \text{ (unpolarized light).}$$



Suppose that the red arrows represent the electric field vectors of some number of equal intensity waves which are heading out of the page:

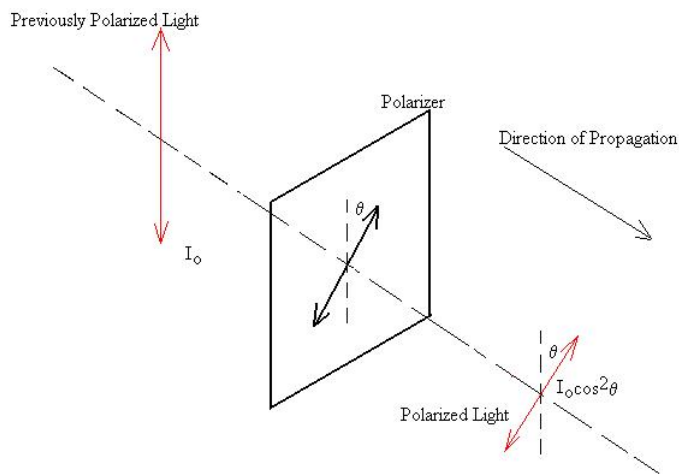


The components that survive the polarizer (that is which are parallel to the axis of the polarizer) are represented by the green lines (the components on the other side can be thought of as just the other halves of the waves already counted). If we add them all up and square the result (remember that $I \sim E^2$), we get a factor of $1/2$. Also, the light exiting the polarizer will be polarized in the direction parallel to the polarization axis of the polarizer.

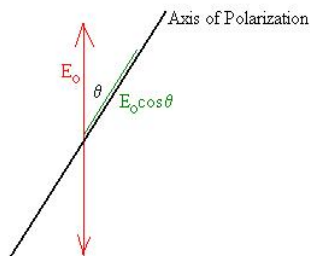
If previously polarized light is incident on an ideal polarizer, the transmitted intensity is given by:

$$I = I_0 \cos^2 \theta,$$

where θ is the angle between the initial plane of polarization and the axis of the polarizer. The transmitted light will be polarized in the same plane as the polarization axis of the polarizer.



The origin of the cosine squared term should be clear to you. Once again, consider the components of the electric field:



Only the component of the original electric field in the direction of the polarizer's axis will be transmitted; that is to say, $E_0 \cos \theta$. Since the intensity goes as E^2 , we obtain the cosine squared term.

Question:

In class, we saw that crossed polarizers allow no light to be transmitted. The first polarizer polarizes the transmitted light along a plane we'll refer to as the 0° plane. The second polarizer has its axis at right angles to this, so the angle θ is 90° ; since $\cos 90^\circ = 0$, there is no light transmitted.

However, we find that when a third polarizer is inserted between the original two, some light does get transmitted. How is this so? How much light gets through? [Click here](#) for the explanation.

A Note for Those Who Care

The optical impedance of a non-conducting material is given by $Z = [\mu_0 / \kappa \epsilon_0]^{1/2} = Z_{\text{vacuum}} / n$. Note the similarities between electromagnetic waves and transverse waves on a string:

	Velocity	Impedance
Wave on a String	$v = [T/\mu]^{1/2}$	$Z = [T\mu]^{1/2}$
EM Wave	$v = [1/\kappa\epsilon_0\mu_0]^{1/2}$	$Z = [\mu_0/\kappa\epsilon_0]^{1/2}$

The same rules regarding the inversion of the reflected waves when encountering an interface are the same as for strings if we consider the magnetic field of the wave; the electric field component follows the reverse, namely inversion on reflection when going from high Z to low Z (low n to high n), and no inversion from low Z to high Z (high n to low n).

A Note for the Truly AR

Unpolarized light really means that the light observed contains equal numbers of waves (which are individually polarized) from many sources, but which when added together exhibit no preferred plane of oscillation.

D Baum - 2002