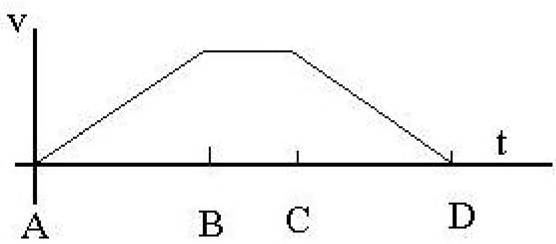
2-4

Draw a velocity *vs* time graph, similar to the one shown below. Note that it may not be to scale! Let v_c be the coasting speed. We know that the entire time interval $t_{total} = t_{AB} + t_{BC} + t_{CD} =$ 5 mins. We also know that the distances AB, BC, and CD are equal. These distances are represented on this type of graph by the areas under the curves. Since the area of each triangle at the ends of the trip must have the same area as the rectangle representing the middle section of the trip, and since the heights of all three areas are the same, the triangular sections must have bases (time intervals) which are twice as long as that of the rectangle. Therefore, **t_{AB} = 2 mins, t_{BC} = 1 min, and t_{CD} = 2 mins**.



This problem is a great example of the inadvisibility of simply plugging-and-chugging equations. A little thinking produces a quick and elegant answer. Let's look at what the algebraic solution looks like:

$t_{AB} + t_{BC} + t_{CD} = 5$ mins
Let d be the distance between adjacent stations, and D be the total distance, so that D = 3d.
Let $a_{AB} = a$, $a_{BC} = 0$, and $a_{CD} = -a$.

For each interval, write the equations $\Delta x = v_o t + {}^1/_2 at^2$ and $v_f = v_o + at$, using the given values:
for AB: $d = 0{*}t_{AB} + {}^1/_2 a(t_{AB})^2$ $\quad v_c = 0 + at_{AB}$ $\quad$ => $d = {}^1/_2 a(t_{AB})^2$ $\quad v_c = at_{AB}$
for BC: $d = v_c t_{BC} + {}^1/_2{*}0{*}(t_{BC})^2$ $\quad v_c = v_c + 0{*}t_{BC}$ $\quad$ => $d = v_c t_{BC}$
for CD: $d = v_c{*}t_{CD} + {}^1/_2(-a)(t_{CD})^2$ $\quad 0 = v_c + (-a)t_{CD}$ $\quad$ => $d = v_c{*}t_{CD} - {}^1/_2 a(t_{CD})^2$ $\quad v_c = at_{CD}$

Since $t_{AB} = v_c/a$ and $t_{CD} = v_c/a$, it seems clear that $t_{AB} = t_{CD}$.
Since $d = {}^1/_2 a(t_{AB})^2$ and $d = v_c t_{BC}$, we can say that ${}^1/_2 a(t_{AB})^2 = v_c t_{BC}$.
Since $v_c = at_{AB}$, we can substitute into the last expression and rearrange:
${}^1/_2 a(t_{AB})^2 = at_{AB}t_{BC}$ => ${}^1/_2 t_{AB} = t_{BC}$

So, our first equation becomes $t_{AB} + t_{BC} + t_{CD} = t_{AB} + 0.5t_{AB} + t_{AB} = 2.5\, t_{AB} = 5$ mins => $t_{AB} = 2$ mins
Therefore, $t_{BC} = {}^1/_2 t_{AB} = 1$ min and $t_{CD} = t_{AB} = 2$ mins.

A somewhat intermediate method might be to observe that the average velocities in the first and third intervals are half that in the second, since

$v_{ave} = [v_f + v_o]/2$,

and the train either begins at rest or ends at rest in these intervals. Assuming constant acceleration, this seems fine, so that

$(v_{ave})_{AB} = {}^1/_2(v_{ave})_{BC} = (v_{ave})_{CD}$.

Since the velocities are half in the outer intervals, but the displacements are the same, the times in the outer intervals must be correspondingly double that of the middle interval. So, we have again that

$t_{AB} + t_{BC} + t_{CD} = t_{AB} + 0.5t_{AB} + t_{AB} = 2.5\ t_{AB} = 5$ mins => $t_{AB} = 2$ mins

Therefore, $t_{BC} = {}^1/_2 t_{AB} = 1$ min and $t_{CD} = t_{AB} = 2$ mins.