3-3)

Pick the origin to be at the bottom of the hill. Positive x is to the right (in the figure) and positive y is up.
Let theta be that angle of launch as measured from the horizontal, and let ø be the angle the hill surface makes with the horizontal.

$v_o = 32$ m/s
$ø = 30^o$
$\theta_i = ?$
D = distance up the hill to the target = 60 m, so,

$x_i = 0$
x_f = the horizontal distance to the target D cosø = 60 cos30^o = 52.96m
$v_{ix} = v_o \cos\theta_o$
$v_{fx} = ?$
$a_x = 0$ (influence of only gravity)
t = ?

$y_i = 0$ m
y_f = D sinø = 30 m
$v_{iy} = v_o \sin\theta_o$
$v_{fy} = ?$
$a_y = -9.8$ m/s^2 (influence of only gravity)

$y_f = y_i + v_{iy}t + {}^1/_2 a_y t^2$
$D \sin ø = 0 + v_o \sin\theta_o t + {}^1/_2 a_y t^2$

$x_f = x_i + v_{ix}t + {}^1/_2 a_x t^2$
$D \cos ø = 0 + v_o \cos\theta_o t + 0$

Solve the x equation for t and substitute:
$t = D\cos ø / v_o \cos\theta_o$
$D \sin ø = 0 + v_o \sin\theta_o [L\cos ø / v_o \cos\theta_o] + {}^1/_2 a_y [D\cos ø / v_o \cos\theta_o]^2$

Simplify by dividing by Lcosø:
$\tan ø = \tan\theta_o + [a_y D\cos ø / 2v_o^2][\cos\theta_o]^{-2}$

Use a trig identity: $1 + \tan^2\theta_o = [\cos\theta_o]^{-2}$
$\tan ø = \tan\theta_o + [a_y D\cos ø / 2v_o^2][1 + \tan^2\theta_o]$
$[a_y D\cos ø / 2v_o^2]\tan^2\theta_o + \tan\theta_o + [a_y D\cos ø / 2v_o^2] - \tan ø = 0$
This is a quadratic equation. Let $A = a_y D\cos ø / 2v_o^2$ and $C = [a_y D\cos ø / 2v_o^2] - \tan ø$

$A\tan^2\theta_o + \tan\theta_o + C = 0$
$\tan\theta_o = [-1 \pm [1^2 - 4AC]^{1/2}/2A$
Since we want the object to move to the +x direction, the angles we find should be between -90^o and +90^o.

OK let's check this out for three cases:

Let the ground be level, or ø = 0. Then
$A = a_yD\cos ø/2v_o^2 = -9.8*60*1/2*32^2 = -0.287$
$C = [a_yD\cos ø/2v_o^2] - \tan ø = -0.287 - 0 = -0.287$
So, $\tan\theta_o = [-1 \pm [1^2 - 4AC]^{1/2}/2A = [-1 \pm [1^2 - 4(-0.287)(-0.287)]^{1/2}/2(-0.287) = 0.316$ or 3.169
These correspond to angles of 17.5^o and 72.59^o, which are, as we saw for the Range equation, complementary.

For an upward incline of ø = 30^o:
$A = a_yD\cos ø/2v_o^2 = -9.8*60*0.866/2*32^2 = -0.249$
$C = [a_yD\cos ø/2v_o^2] - \tan ø = -0.249 - 0.577 = -0.826$
So, $\tan\theta_o = [-1 \pm [1^2 - 4AC]^{1/2}/2A = [-1 \pm [1^2 - 4(-0.249)(-0.826)]^{1/2}/2(-0.249) = 2.854$ or 1.163
These correspond to angles of 49.3^o and 70.7^o. Note that these are not complements, since the terrain was not flat.

For a downward incline of ø = -30^o:
$A = a_yD\cos ø/2v_o^2 = -9.8*60*0.866/2*32^2 = -0.249$
$C = [a_yD\cos ø/2v_o^2] - \tan ø = -0.249 + 0.577 = 0.328$
So, $\tan\theta_o = [-1 \pm [1^2 - 4AC]^{1/2}/2A = [-1 \pm [1^2 - 4(-0.249)(0.328)]^{1/2}/2(-0.249) = -0.305$ or 4.321
These correspond to angles of -17.0^o and 77.0^o. Note that these are not complements, since the terrain was not flat.