6-4)

Let x be the distance of the asteroid from the center of the earth. We're given that $F_G = Cx^{-2}$ and that when $x = R_E$, $F_G = gm$. Here, g is specifically the strength of the gravitational field at the earth's surface.

So at the surface of the earth, $F_G(R_E) = gm = CR_E^{-2}$ -> $C = gmR_E^2$, so $F(x) = gm\, R_E^2\, x^{-2}$. Work done by the gravitational force on an object of mass m is then

$$W_G = \int_{x_i}^{x_f} \boldsymbol{F}_G(x) \cdot d\boldsymbol{x}\,.$$

Now since $d\mathbf{x}$ points outward and $\mathbf{F}_G$ points inward, there is a cos(180°) in the dot product.

$$W_G = -\int_{x_i}^{x_f} F_G(x)\, dx\,.$$

We'll start the asteroid out infinitely far from the earth:

$$W_G = \Delta K = -\int_{\infty}^{R_E} gmR_E^2 x^{-2}\, dx\,.$$

$$K_f = K_i - gmR_E^2 \int_{\infty}^{R_E} x^{-2}\, dx\,.$$

$$K_f = K_i - gmR_E^2\left[-x^{-1}\Big|_{\infty}^{R_E}\right]$$

$$K_f = K_i + gmR_E^2\left[\frac{1}{R_E} - \frac{1}{\infty}\right] = K_i + gmR_E$$

b & c)

The minimum KE will be when $K_i = 0$,

$$K_{fMIN} = gmR_E = 10 \times 20{,}000 \times 6{,}400{,}000 = 1.3{\times}10^{12}\ J,$$

or about 1/50th the energy of the Hiroshima explosion.

a)

$$K_{f\,MIN} = \tfrac{1}{2}mv_{f\,MIN}^2 = gmR_E.$$

$$v_{f\,MIN} = \sqrt{\frac{2K_{f\ MIN}}{m}} = \sqrt{\frac{2 \times 1.3 \times 10^{12}}{20{,}000}} = 11{,}314\,\frac{m}{s}\,.$$

This is actually the reverse of the concept of *escape velocity*. How quickly would something have to be launched from the surface of the earth to escape from the earth's gravitational well?

v_{ESCAPE} = 11.3 km/s.