8-8)

Use Conservation of Mechanical Energy:

 $W_{\rm NC} = \Delta K + \Delta U$

No non-conservative forces do work: the normal force is perpendicular to the direction of motion, and we've already discussed how the translational work due to friction cancels the rotation work of friction if the object rolls without slipping.

 $0 = K_{ROTf} - K_{ROTi} + K_{TRANSf} - K_{TRANSi} + U_{Gf} - U_{Gi}$

So, if we let the starting position correspond to $y_i = h$ and the top of the loop-de-loop be $y_f = 2R$,

$$0 = \frac{1}{2}mv_{CMf}^{2} + \frac{1}{2}I\omega_{f}^{2} + gmy_{f} - gmy_{i}$$

 $v_{CM} = R\omega$

 $0 = {}^{1}\!/_{2}mv_{CM}^{2} + {}^{1}\!/_{2}({}^{2}\!/_{3}mR^{2})(v_{CM}/R)^{2} + gmy_{f} - gmy_{i}$

 $gm(h-2R) = \frac{1}{2}mv_{CM}^{2} + \frac{1}{3}mv_{CM}^{2}$

 $g(h-2R) = \frac{5}{6VCM^2}$

Now, if the object is just to make it over the loop (a problem we've done before, 5-8), there needs to be a minimum speed at the top of the loop. Consider Newton's second law for the object at the top of the loop:

 $N + mg = ma_C = mv^2/R$

The minimum speed is given when N goes to zero: $mg = mv^2/R$ or $mgR = mv^2$ Substitute this requirement back in to the energy equation:

 $g(h - 2R) = \frac{5}{6}mgR$

And so

 $h_{min} = [2 + \frac{5}{6}]R = \frac{\frac{17}{6}R}{6}$